Electric-field-induced resonant spin polarization in a two-dimensional electron gas

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Electric response of spin polarization in two-dimensional electron gas with structural inversion asymmetry or Rashba spin-orbit coupling subjected to a magnetic field was studied by means of the linear response theory and numerical simulation with the disorder effect. It was found that an electric resonant response of spin polarization occurs when the Fermi surface is located near the crossing of two Landau levels, which is induced from the competition between the spin-orbit coupling and Zeeman splitting. The scaling behavior was investigated with a simplified two-level model, and the height of the resonant peak is reciprocally proportional to the electric field at low temperatures and to temperature for finite electric fields, respectively. Finally, numerical simulation illustrated that impurity potential opens an energy gap near the resonant point and suppresses the effect gradually with increasing strength of disorder. This resonant effect may provide an efficient way to control spin polarization by an external electric field.

I. INTRODUCTION

Recently, electric or nonmagnetic generation of spin polarization in semiconductors has attracted a lot of interest because of its potential application in spintronic devices of semiconductors. In this attempt, the motion of electron interacts with its spin via the spin-orbit coupling, which provides a possible way to control electron spin by electric field instead of magnetic field. Several experiments were resolved to produce spin polarization in quantum wells, strained semiconductors, and hole-doped heterojunction. In a two-dimensional electron gas (2DEG) with structural inversion asymmetry, it was understood that the spin-orbit coupling generates an effective momentum-dependent field to induce a net bulk spin polarization by electric fields or currents. Recently, it was also proposed to generate spin polarization in a bulk 2DEG in the presence of in-plane magnetic fields and electric fields. Electric-induced spin accumulation near the boundary of a sample was already observed experimentally in either n- or p-doped semiconductors as a consequence of spin Hall effect. The ac field was also applied to induce and detect spin polarization. The spin orientation was achieved by the excitation of a high-frequency electric field. Spin resonance by an ac field was also discussed extensively.

Generally speaking, we may introduce the electric spin susceptibility \( \chi_{E}^{\alpha\beta} \) to describe the response of spin polarization \( S_{\alpha} \) to an external electric field \( E_{\beta} \):

\[
S_{\alpha}(E) = S_{\alpha}(0) + \chi_{E}^{\alpha\beta} E_{\beta},
\]

where \( S_{\alpha}(0) \) is the spin polarization in the absence of electric field. Usually, the electric spin susceptibility is a tensor, not a vector. The spin polarization is determined by the symmetry of spin-orbit coupling of the system. To generate spin polarization efficiently, a large electric spin susceptibility is expected. In this paper, we propose an electric resonant response of spin polarization in 2DEG with Rashba spin-orbit coupling. First, the linear response theory shows that the electric spin susceptibility becomes divergent when the crossing point of two Landau levels is accidentally located near the Fermi surface. The additional degeneracy of two Landau levels is attributed to competition between the spin-orbit coupling and the Zeeman splitting. Then, a simplified two-level model was proposed to investigate the scaling behaviors of the resonant electric spin susceptibility. The resonant values of the electric spin susceptibility decay with either the applied electric field at low temperatures or with the temperatures in a weak electric field. Finally, we take into account the disorder effect and apply the truncation approximation to study the system numerically, which goes beyond the linear response theory. The dependence of the divergent behavior on the electric field and temperature was presented for finite disorder strengths. It is damped with the electric field and temperature when the energy scale of the electric field and temperature is larger than the impurity potential. The numerical consequence is consistent with the result of linear response theory.

II. MODEL HAMILTONIAN AND LINEAR RESPONSE THEORY

We start with a 2DEG with Rashba coupling confined in a two-dimensional plane \( L_x \times L_y \). The model Hamiltonian in the presence of a perpendicular magnetic field \( B \) is given by

\[
H = H_0 + H_R,
\]

where

\[
H_0 = \frac{1}{2m^*} \vec{\Pi} \cdot \vec{\Pi} - \frac{1}{2} g_s \mu_B B \sigma_z,
\]

and the Rashba coupling

\[
H_R = \frac{\alpha}{\hbar} (\Pi_x \sigma_y - \Pi_y \sigma_x).
\]

\( m^* \) is the effective mass of the electron, \( \vec{\Pi} = \vec{p} + \frac{\alpha}{\hbar} \vec{A} \) is the kinetic momentum, \( g_s \) is the Lande g factor, and \( \mu_B \) is the Bohr magneton.

We take the Landau gauge \( A = y B \hat{x} \) and consider the periodic boundary condition in the \( \hat{x} \) direction. The Hamiltonian can be solved analytically with the eigenvalues

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\[ e_{n,s} = \hbar \omega_c \left[ n + \frac{s}{2} \sqrt{(1-g)^2 + 8n \eta_R^2} \right], \tag{5} \]

where \( n \) is a non-negative integer and \( s = \pm 1 \). Here, the cyclotron frequency \( \omega_c = eB/m^*c \), the magnetic length \( l_B = \sqrt{\hbar c/eB} \), the dimensionless \( g \) factor \( g = g_s m^*/2m_c \), and the effective coupling \( \eta_R = \alpha/l_B \hbar \omega_c \). The corresponding eigenvalues are expressed as

\[ |nks⟩ = \left( \frac{ic_{ns}^+ \phi_{nk}}{−sc_{ns}^+ \phi_{ns−1k}} \right), \tag{6} \]

where \( \phi_{nk} \) is the eigenstate of the \( k \)th Landau level with \( k = p_n \), the good quantum number because of the periodic boundary condition along the \( x \) direction. \( c_{ns}^+ \) are the coupling parameters of spins up and down. \( c_{ns}^+ = 1 \), \( c_{ns}^- = 0 \), and \( c_{ns}^0 = 1/\sqrt{1 + (u_n + s \sqrt{1 + u_n^2})^2} \) with \( u_n = (1−g)/\sqrt{8n \eta_R} \) for \( n \geq 1 \). Each Landau level has a degeneracy \( N_k = L_{xx}/2 \pi c_n^2 \). One of the remarkable features of the spectra is the additional crossing of Landau levels, which is generated by the competition between Rashba coupling and Zeeman splitting such as \( e_{n,ss+1} = e_{n,s+1} = e_{n+1,s} = 1 \) if the integer \( n \) and the magnetic field \( B \) satisfy

\[ \sqrt{(1-g)^2 + 8n \eta_R^2} + \sqrt{(1-g)^2 + 8(n+1) \eta_R^2} = 2. \tag{7} \]

One important factor in this system is the filling factor, i.e., the ratio of number of charge carriers to the Landau degeneracy \( \nu = N_e/N_s = 2 m^2 c_l n_c \) (\( n_c \) is the density of charge carriers). For a specific density, the filling factor is proportional to \( 1/B \), and for a specific field, it is proportional to the density \( n_c \).

With this solution in mind, we are ready to study the electric response of spin polarization when the Fermi surface is located near the resonant point. We apply an \textit{in-plane} weak electric field, say, \( E_y \) along the \( y \) direction. Then the electric spin susceptibility can be evaluated by means of the Kubo formula in the weak field limit.\(^{23} \)

\[ \chi^{yy}_{E} = \frac{\epsilon c_n^2 \hbar}{L_{xx}L_{yy}} \text{Im} \sum_{n'n'k'k} \left( (e_{n's'} - e_{n's}) (e_{n's'} - e_{n's}) + i\hbar/\tau \right) \times (nks|S_{nn'k'k'}|^2)(n'k's'|v_y|nks), \tag{8} \]

where \( v_y \) is the velocity in the \( y \) direction, \( f_{ns} \) are the Fermi-Dirac distribution functions, and \( \tau \) can be regarded as the lifetime of the quasiparticles. From this equation, the denominator becomes singular when two energy levels are degenerate near the Fermi surface. As a result, it is possible that the electric spin susceptibility may become singular for a long lifetime \( \tau \to \infty \), i.e., \( \chi^{yy}_{E} \) may become divergent. For a finite \( \tau \) or finite temperature, \( \chi^{yy}_{E} \) still remains finite.

For the purpose of numerical calculations, without losing generality, we take the model parameters such that the lowest energy level \( |1⟩ = |n=0, k=0, s=+1⟩ \) and the first excited level \( |2⟩ = |n=1, k=1, s=−1⟩ \) cross at a critical magnetic field \( B = 9.8 \) T. In the clean limit, i.e., \( \tau \to \infty \), we plot \( \chi^{yy}_{E} \) versus the inverse of the magnetic field \( 1/B \) in Fig. 1 for a fixed electron density \( n_c \). It is found that a resonant peak occurs at the critical magnetic field corresponding to a filling factor \( \nu = 0.5 \). The resonant peak indicates that it is possible for a weak electric field to induce a finite spin polarization at the resonant point.

It is worth stressing that the Kubo formula is a result of perturbation and is valid only when \( E \to 0 \) (but \( E \neq 0 \)). At a first glance, the divergence of spin susceptibility in Fig. 1 may be unphysical because it might be caused by the approximation of perturbation in the linear response theory. To clarify the problem, we have to go beyond the linear response theory and investigate the problem by means of the nonperturbative approach.

### III. Nonlinear Behaviors in Simplified Two-Level Model

In order to understand the physical origin of the resonance and to analyze the scaling behaviors of \( \chi^{yy}_{E} \) at or near the resonant point, we consider a simplified two-level model around the resonant point. This is based on the fact that the dominant contribution to the electric-field-induced resonance is attributed to the energy crossing of the two levels. This truncation approximation reduces an infinite-dimension problem into a two-level one. As the two-level problem can be solved analytically, the effect of electric field can be taken into account. The method goes beyond the perturbation method and the nonlinear response of spin polarization to an external field can be revealed. For this purpose, we are only concerned with the two nearly degenerated levels \( |1⟩ \) and \( |2⟩ \) and ignore the contribution from other levels of higher energies. The Landau degeneracy of the two states for different \( k \) will be taken into account at the last step to calculate physical quantities since the Hamiltonian in an external field is diagonalized in block in the \( k \) space. Then the total Hamiltonian including the electric potential can be reduced to a \( 2 \times 2 \) matrix:
Near the resonant point $\varepsilon \to 0$, we have $\chi_E^{yy} \propto 1/E$ at low temperatures $kT \ll |v_E|$, meanwhile, $\chi_E^{yy} \propto 1/T$ for a weak electric field $kT \gg |v_E|$. It depends on the energy scales of the electric field energy $\varepsilon$ and temperatures $T$. In Figs. 3(b) and 3(c), the dependence of the peak value of $\chi_E^{yy}$ on the electric field and temperature is plotted.
From the point of view of spin polarization, the spin polarization per unit area is

$$S_y(E) = \frac{\hbar f_-(1-f_+)}{2} \frac{|v_0|}{\sqrt{v^2 + |v_0|^2}} (1-e^{-2\sqrt{v^2 + |v_0|^2}/kT}).$$

(12)

At the resonant point $\epsilon=0$ and in low temperatures, $kT/|v_0|\rightarrow 0$, $S_y(E) = \hbar f_-(1-f_+)/(2\pi l_B^2 \nu)$. Thus, a finite spin polarization can be induced by a weak electric field. In the absence of electric field,

$$S_y(0) = \frac{(S^+ f_+ + S^- f_-)_{E=0}}{2\pi l_B^2} = 0.$$  

(13)

It is worthy to notice that even in the weak field limit, $\lim_{E\rightarrow 0} S_y(E) \neq S_y(0)$ at the resonant point. This is the key point to understand this resonant effect. The finite spin polarization induced by a small electric field indicates the non-linear behaviors of this effect. As a result, the spin susceptibility becomes singular in $1/E$ as $E\rightarrow 0$. These results are consistent with the prediction of the Kubo formula in Fig. 1. It indicates that the resonance is not caused by the perturbation approximation in the linear response approach but is attributed to the removal of the degeneracy of two crossing Landau levels by the external field.

IV. NUMERICAL SIMULATION

After establishing a physical picture for the electric resonance of spin polarization in a clean limit, we turn to study the effect of impurities. The effect of impurities can be described by introducing a finite lifetime $\tau$ phenomenologically in the Kubo formula. To go beyond the linear response theory, we do numerical simulation to study the impurity effect in the real space. For this purpose, we still take the periodic boundary condition in the x direction but an infinite potential wall in the y direction: $V(y) = 0$ for $|y|<L_y/2$, and $+\infty$ otherwise. The disorder potential $U(x,y)$ is taken to be the short-range impurities of strength $u_i$ uniformly distributed at $(x_i,y_i)$ in the plane:

$$U(x,y) = \sum_i u_i \delta(x-x_i) \delta(y-y_i),$$

(14)

where $u_i \in (-u/2,u/2)$, $x_i \in (-L_x/2,L_x/2)$, and $y_i \in (-L_y/2,L_y/2)$. In the absence of impurity potential and Rashba coupling, the confined Landau levels $|\varphi_{nks}\rangle$ have been obtained analytically,

$$|\varphi_{nks}\rangle = \exp(-iy_0m_0^2)\phi_n(y-y_0)/\sqrt{2\pi l_B^2},$$

(15)

where $\phi_n(y-y_0)$ are the confluent hypergeometric functions and $y_0 = 2\pi l_B^2/kL_x$. $\nu_n$ is determined numerically by the boundary condition $|\varphi_{nks}\rangle_{y=L_y/2}=0$. The finite size effect removes the Landau degeneracy near the edge. All the confined Landau levels can be regarded as a complete set of basis. On this basis, the kinetic energy and the Zeeman term, $H_0$, has been diagonalized. The elements of the Rashba coupling and the disorder potential are $\langle \varphi_n|e\tau|\varphi_i\rangle$ and $\langle \varphi_n|x_i|\varphi_i\rangle$, respectively. After taking into account the impurity potential and Rashba coupling, we perform the truncation approximation to reduce the whole Hamiltonian into an effective one with finite dimension. Furthermore, the effective Hamiltonian will be diagonalized numerically to calculate the eigenvalues and eigenfunctions. In the calculations, we take $n_{max}+1$ Landau levels, $n=0,1,\ldots,n_{max}$, and each Landau level has $N_k$ discrete values of $k (n=0,1,2,\ldots,N_k-1).$ 

$N_k$ can also be expressed as the maximum number of electrons accommodated in each Landau level. Then the number of basis functions we retained in the truncation approximation is $N=2\times (n_{max}+1)\times N_k$ with double degeneracy of spin. By diagonalizing the $N\times N$ Hamiltonian numerically, one gets $N$-eigenvalues $E_n^N$ and $N$-component wave vectors $|\Psi_{n}\rangle = \sum_{m=0}^{N_k} a_{nks}\langle \varphi_{mks}|$. which are the superposition of the basis $|\varphi_{nks}\rangle$. When a weak electric field $V=eE_y$ is applied in the y direction, the expectation value of

FIG. 3. (Color online) (a) The spin susceptibility $\chi^{\alpha}_{yy}$ as a function of $1/B$ near the resonant point for different electric fields at $T=10^{-6}$ K. (b) The electric field $E$ dependence of the peak value of $\chi^{\alpha}_{yy}$ at a low temperature $T=10^{-6}$ K. (c) The temperature $T$ dependence of the peak value for an electric field $E=10^{-6}$ N/C. The unit of $\chi^{\alpha}_{yy}$ is $\hbar C/4\pi l_B^2 N$ and that of $B$ is T.
the total spin can be calculated numerically. As a result, $\chi_{E}^{yy}$ can be obtained,

$$\chi_{E}^{yy} = \frac{1}{L_xL_y} \sum_{m} \frac{\delta(S_y)_m}{E_y},$$

(16)

where

$$\delta(S_y)_m = f_m(\Psi_m^{N}|S_y^{N})_{E} - f_m(\Psi_m^{N}|S_y^{N})_{E=0},$$

(17)

with $f_m$ the Fermi-Dirac distribution.

In our calculations, the model parameters are taken as $L_x/N_1=L_y/N_2=\sqrt{2} \pi \ell_b$. In this paper, we take $N_1=10$ and $N_2=6$. Then the maximum number of electrons at each level is $N_y=L_y/N_1=60$. The number of Landau levels are truncated to $n_{max}=5$; hence, $N=720$. In this truncation approximation, we are concerned only with the low energy physics. The magnetic field is chosen as $B=9.8$ T such that $E_{0,1}=E_{1,-1}$ for the lowest and first excited level in the bulk region, with other parameters the same as in Fig. 1. 140 impurities of relative strength $|\lambda|=|u_i/(2\pi \ell_b \hbar \omega)| \leq \lambda$ are randomly distributed over the sample. For each configuration of impurities, both the strength and position are generated randomly. The states are filled from lower to higher energy, and correspondingly, $\chi_{E}^{yy}$ can be calculated by the formula in Eq. (16) for each configuration.

After averaging over $10^4$ different impurity configurations, we plot the average value of $\chi_{E}^{yy}$ versus the filling factor $\nu$ in Fig. 4(a) for different strengths of disorder, $\lambda$. The temperature is set to $T=0.025$ K, hence, the ratio $kT/\hbar \omega_i = 2 \times 10^{-6} \leq \lambda$ and the electric field $E = 10^{-6}$ N/C such that $\nu = eE_1/\hbar \omega_1 \approx 10^{-8} \lambda$. When the electrons are filled up to $\nu = 0.5$, $\chi_{E}^{yy}$ displays a resonant peak. In contrast, $\chi_{E}^{yy}$ is finite in the nondegenerate region with the filling number $\nu = 2-4$ and tends to be suppressed when $\lambda = 1.6 \times 10^{-3}$. The relative error due to impurity fluctuation is estimated to be around 0.01% and up to 5% at the resonant point due to the fact that the susceptibility is very sensitive to the energy gap opened by impurities. The peak height decreases with the disorder strength $\lambda$ and the peak value versus $\lambda$ is demonstrated in the inset. When we extrapolate to the limit $\lambda \to \infty$, the peak value tends to be suppressed completely. The dependence of $\chi_{E}^{yy}$ on the disorder strength is similar to that on the electric field, thus, the impurity scattering opens a gap between the degenerate levels just like the electric field. Inversely, we fix the disorder strength at $\lambda = 10^{-5}$ and plot the electric spin susceptibility $\chi_{E}^{yy}$ in Fig. 4(b) at different temperatures $T=kT/\hbar \omega_1 > \lambda$. The peak height of $\chi_{E}^{yy}$ decays with temperatures. The scaling behavior of the peak value versus $T$ is shown in the inset. In the presence of disorder, the dependence of the resonant peak of $\chi_{E}^{yy}$ on the electric field is also simulated and plotted in Fig. 5, denoted by the black and red dots respectively for the disorder strengths $\lambda = 10^{-5}$ and $\lambda = 10^{-3}$. The peak value $\chi_{E}^{yy}$ at different strengths of disorder $\lambda$ is plotted in Fig. 4(b) at different temperatures $T$. The spin polarization $S_y$ is denoted, respectively, by the dots and diamonds in the presence of disorder. The temperature is set to zero. The electric spin susceptibility $\chi_{E}^{yy}$ is in units of $\hbar C/4 \pi I_d^2 N$ and $S_y$ is in unit of $\hbar/4 I_d^2$.

FIG. 4. (Color online) (a) $\chi_{E}^{yy}$ versus filling factor $\nu$ for four different disorder strengths $\lambda$ for a fixed temperature $T=0.025$ K. The inset shows the disorder strength dependence of the peak value. (b) $\chi_{E}^{yy}$ versus the filling factor $\nu$ at different temperatures $T$ for disorder strength $\lambda = 10^{-5}$. The inset is for the temperature $T$ dependence of the peak value. The electric field is taken to be as low as $E_1=10^{-8}$ N/C and the spin susceptibility is in units of $\hbar C/4 \pi I_d^2 N$.

FIG. 5. (Color online) The electric field $E$ dependence of $\chi_{E}^{yy}$ and the spin polarization $S_y$ are denoted, respectively, by the dots and diamonds in the presence of disorder. The temperature is set to zero. The electric spin susceptibility $\chi_{E}^{yy}$ is in units of $\hbar C/4 \pi I_d^2 N$ and $S_y$ is in unit of $\hbar/4 I_d^2$.
$\lambda=10^{-5}$ and $10^{-4}$. At low temperatures, when $\bar{e}=eE,\epsilon_f/\hbar \omega_c \ll \lambda$, $\chi_E^{xx}$ is independent of the electric field $E$ but diverges as $1/E$ when it is comparable with or greater than the disorder strength $\lambda$. The spin polarization $S_z(E)=S_z(0)+\chi_E^{xy}E$ increases linearly with a weaker electric field but saturates at higher electric fields, as plotted in Fig. 5 by the black and red diamonds for the disorder strengths $\lambda=10^{-5}$ and $10^{-4}$, respectively.

V. SUMMARY AND DISCUSSION

At last, the occurrence of this resonance is not limited only in the Rashba system. The electric spin susceptibility depends on the symmetry of spin-orbit coupling explicitly. The present work can be generalized to a system with the Dresselhaus coupling $H_D=\beta(p,\sigma_r-p,\sigma_r)$. Because the Rashba coupling can be transformed to the Dresselhaus coupling under the transformations of $\sigma_r\rightarrow\sigma_x$, $\sigma_y\rightarrow\sigma_y$, and $\sigma_z\rightarrow-\sigma_x$, we conclude that it is $\chi_E^{xy}$ instead of $\chi_E^{xy}$ which would become divergent at the resonant point.

In conclusion, a tiny electric field may generate a finite spin polarization in a disordered Rashba system in the presence of a magnetic field. As a result, the electric spin susceptibility exhibits a resonant peak when the Fermi surface goes through the crossing point of two Landau levels. Numerical results demonstrate that the result goes beyond the linear response theory. This provides a mechanism to control spin polarization efficiently by an electric field in semiconductors. As the spin polarization can be measured very accurately, it is believed that the effect can be verified in the samples of 2DEG, such as the heterojunction of InGaAs/InAlAs.

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20. In the linear response theory, the spin susceptibility is also defined as $\chi^{xx}_E=\partial S_z/\partial E|_{E=0}$. In this paper, due to the nonlinear behavior of $\chi^{xx}_E$ to external field $E$, we use the definition in Eq. (1) for a finite field.