Realization of a Special Class of Admittances with One Damper and One Inerter

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Passive Networks

Definition

A network is passive if the behaviors of its terminals satisfies:

\[ \int_{-\infty}^{T} I(t) V(t) \, dt \geq 0, \]

for all the physically acceptable current \( I(t) \), voltage \( V(t) \), and all \( T \).

**Definition**

$Z(s)$ is defined to be a *positive-real function* if $Z(s)$ is analytic and $\Re Z(s) \geq 0$ in $\Re [s] > 0$. 

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The admittance (or impedance) of a passive network must be positive-real, i.e.

$$\Re(Y(j\omega)) \geq 0 \text{ for all } \omega.$$ 

(Not too difficult to prove.)
Defining admittance as $Y(s) = I(s)/V(s)$, and impedance as $Z(s) = V(s)/I(s)$, we can obtain the following conclusion.

**Theorem**

The impedance (admittance) of any passive network must be a positive-real function.

What about the converse?

In 1931, O. Brune established a procedure that can realize any positive-real function with passive resistors, inductors, capacitors, and transformers.

A minimum function $Z(s)$ is a positive-real function with no poles and zeros on $j\mathbb{R} \cup \{\infty\}$ and the real part of $Z(j\omega)$ equal to 0 at one or more frequencies.
Foster Preamble for a Positive-Real Function

Removal of poles on \( j\mathbb{R} \cup \{\infty\} \)

\[
Z = sL + Z_1, \quad L > 0.
\]

Removal of zeros on \( j\mathbb{R} \cup \{\infty\} \)

\[
Z = \left( \frac{2K_i s}{s^2 + \omega_i^2} + Y_1 \right)^{-1}, \quad K_i > 0.
\]

Subtract minimum real part

\[
Z = R + Z_2, \quad R > 0.
\]

Foster Preamble can yield any positive-real function to a minimum one.
Hence for any minimum function $Z(s)$, we have

$$L_1 < 0 \quad \quad L_3 > 0 \quad \quad L_2 > 0 \quad \quad C_2 > 0 \quad \quad Z_3(s)$$

It can be proven that $Z_3(s)$ is positive-real, $\delta[Z_3(s)] < \delta[Z(s)]$ and $L_1 L_3 < 0$. However, the negative inductor can be absorbed in transformers.

$$L_p = L_1 + L_2 > 0, \quad L_s = L_2 + L_3 > 0, \quad M = L_2 > 0.$$
Since $\delta[Z_3(s)] < \delta[Z(s)]$ and $Z_3(s)$ is positive-real after a Brune cycle. Therefore, we conclude that

\[ \text{Brune Synthesis} = \text{Foster Preambles} + \text{Brune Cycles} \]

can realize any positive-real function with passive resistors, inductors, capacitors and transformers.

constructive procedure
R. Bott and R. J. Duffin established a new realization procedure for any positive-real function without transformers.

Bott-Duffin Cycle

Theorem (Richards’s Transformation)

If $Z(s)$ is positive-real, then $R(s) = \left(kZ(s) - sZ(k)\right)/(kZ(k) - sZ(s))$ is positive-real for any $k > 0$, and $\delta[R(s)] \leq \delta[Z(s)]$.

\[ \delta[R_1] = \delta[R_2] < \delta[Z], \text{ where } Z(s) \text{ is a minimum positive-real function.} \]

Redundant elements Exist
Bott-Duffin Synthesis = Foster Preambles + Bott-Duffin Cycles can realize any positive-real function using finite number of resistors, capacitors, and inductors.

We call the resistor, capacitor, and inductor as the three basic two-terminal electrical elements.
Force-Current Analogy:

\[
\text{current} \leftrightarrow \text{force} \\
\text{voltage} \leftrightarrow \text{velocity} \\
\text{electrical ground} \leftrightarrow \text{mechanical ground}
\]

Passivity:

\[
\int_{-\infty}^{T} F(t) v(t) dt \geq 0
\]

Mechanical Immittances:

\[
Z(s) = \hat{v}/\hat{F}, \quad Y(s) = \hat{F}/\hat{v}
\]
Z(s) must be positive-real.

But is \( Z(s) \) always realizable physically?
\[ v = Ri \quad \text{resistor} \leftrightarrow \text{damper} \quad cv = F \]
\[ v = L \frac{di}{dt} \quad \text{inductor} \leftrightarrow \text{spring} \quad kv = \frac{dF}{dt} \]
\[ C \frac{dv}{dt} = i \quad \text{capacitor} \leftrightarrow \text{mass} \quad m \frac{dv}{dt} = F \]

**Analogy of Basic Elements (Force-Current Method)**

What are the terminals of the mass element?

Electrical

Mechanical

MTNS2006, Kyoto, 24 July, 2006
The (ideal) inerter is a two-terminal mechanical device with the property that the equal and opposite force applied at the terminals is proportional to the relative acceleration between them, that is, $F = b \dot{v}$ where $\dot{v} = \dot{v}_1 - \dot{v}_2$.

Applications of the Inerter

1. Vehicle suspension systems control
2. Motorcycle steering control
3. Building systems control
4. Train suspension systems control
5. ... ... ... (Future)
The Missing Mechanical Circuit Element

Michael Z.Q. Chen, Christos Papageorgiou, Frank Scheibe, Fu-Cheng Wang, and Malcolm C. Smith

Abstract

In 2008, two articles in Autosport revealed details of a new mechanical suspension component with the name "J-damper" which had entered Formula One Racing and which was delivering significant performance gains in handling and grip. From its first mention in the 2007 Formula One "spy scandal" there was much speculation about what the J-damper actually was. The Autosport articles revealed that the J-damper was in fact an "inerter" and that its origin lay in academic work on mechanical and electrical circuits at Cambridge University. This article aims to provide an overview of the background and origin of the inerter, its application, and its intimate connection with the classical theory of network synthesis.

The theory of passive electrical networks can be completely transplanted into mechanical networks.

Mechanical Networks Synthesis and Existing Problems

Theory of passive network synthesis

The interest of the investigation is renewed

Passive mechanical systems design

Redundant elements limit its practical use
Outline

1 Introduction
2 Problem Formulation
3 Main Results
4 Conclusion
A Class of Functions to be Investigated

Consider a class of admittance functions in the form of

\[ Y(s) = k \frac{a_0 s^2 + a_1 s + 1}{s(d_0 s^2 + d_1 s + 1)}, \]  

where \( a_0, a_1, d_0, d_1 \geq 0 \), and \( k > 0 \). The mechanical admittances of many suspension struts are always of this form.

**Theorem**

\( Y(s) \) in the form of (1) where \( a_0, a_1, d_0, d_1 \geq 0 \), and \( k > 0 \) is positive-real, if and only if

\[ a_0 d_1 - a_1 d_0 \geq 0, \ a_0 - d_0 \geq 0, \ a_1 - d_1 \geq 0. \]


It is firstly pointed out in [1] that any positive-real $Y(s)$ in this form is realizable as

![Diagram of mechanical network](https://example.com/diagram.png)

by Brune Synthesis (Foster Preamble).

Problem

Consider any positive-real admittance function \( Y(s) \) in the form:

\[
Y(s) = k \frac{a_0 s^2 + a_1 s + 1}{s(d_0 s^2 + d_1 s + 1)},
\]

where \( a_0, a_1, d_0, d_1 \geq 0 \), and \( k > 0 \). What is the necessary and sufficient condition for \( Y(s) \) to be realizable with one damper, one inerter, and an arbitrary number of springs? What about the covering configurations?
Outline

1. Introduction
2. Problem Formulation
3. Main Results
4. Conclusion
Consider a class of admittance functions in the form of

\[ Y(s) = k \frac{a_0 s^2 + a_1 s + 1}{s(d_0 s^2 + d_1 s + 1)}, \]

where \( a_0, a_1, d_0, d_1 \geq 0 \), and \( k > 0 \). \( Y(s) \) is positive-real if and only if

\[ a_0 d_1 - a_1 d_0 \geq 0, \]
\[ a_0 - d_0 \geq 0, \]
\[ a_1 - d_1 \geq 0. \]
A Degenerate Case

Let \( p(s) = a_0 s^2 + a_1 s + 1 \) and \( q(s) = d_0 s^2 + d_1 s + 1 \), then it is known from [1] that the resultant of \( p(s) \) and \( q(s) \) can be calculated as

\[
R_k := (a_0 - d_0)^2 - (a_0 d_1 - a_1 d_0)(a_1 - d_1).
\]

If \( R_k = 0 \), then \( Y(s) \) reduces to

\[
Y(s) = k \frac{as + 1}{s(ds + 1)} = \frac{k}{s} + \frac{1}{\frac{ds}{k(a-d)} + \frac{1}{k(a-d)}},
\]

where \( a \geq d \geq 0 \). Hence \( Y(s) \) can be realized by a network consisting of at most one damper and two springs.

A positive-real function $Y(s)$ can be realized as the driving-point admittance of a network comprising one damper, one inerter, and arbitrary number of springs but no levers if and only if $Y(s)$ can be written in the form of

$$Y(s) = \frac{(R_2 R_3 - R_6^2)s^3 + R_3 s^2 + R_2 s + 1}{s \left( \det R s^3 + (R_1 R_3 - R_5^2)s^2 + (R_1 R_2 - R_4^2)s + R_1 \right)},$$

where $R$ in the form of

$$R = \begin{bmatrix} R_1 & R_4 & R_5 \\ R_4 & R_2 & R_6 \\ R_5 & R_6 & R_3 \end{bmatrix} \quad (2)$$

is non-negative definite, and satisfies either Condition A or Condition B.
Condition A

There exists a first- or second-order minors of $R$ being zero.

Condition B

1. $R_4 R_5 R_6 < 0$;
2. $R_4 R_5 R_6 > 0$, $R_1 > (R_4 R_5 / R_6)$, $R_2 > (R_4 R_6 / R_5)$ and $R_3 > (R_5 R_6 / R_4)$;
3. $R_4 R_5 R_6 > 0$, $R_3 < (R_5 R_6 / R_4)$, and $R_1 R_2 R_3 + R_4 R_5 R_6 - R_1 R_6^2 - R_2 R_5^2 \geq 0$;
4. $R_4 R_5 R_6 > 0$, $R_2 < (R_4 R_6 / R_5)$, and $R_1 R_2 R_3 + R_4 R_5 R_6 - R_1 R_6^2 - R_3 R_4^2 \geq 0$;
5. $R_4 R_5 R_6 > 0$, $R_1 < (R_4 R_5 / R_6)$, and $R_1 R_2 R_3 + R_4 R_5 R_6 - R_3 R_4^2 - R_2 R_5^2 \geq 0$.

Covering Configurations

\begin{align*}
&k_1 \quad k_3 \quad \text{c} \quad k_1 \\
&k_4 \\
\end{align*}

\begin{align*}
&k_2 \\
&k_3 \quad k_4 \\
\end{align*}

\begin{align*}
&k_1 \\
&k_3 \quad k_4 \\
\end{align*}

\begin{align*}
&k_2 \\
&k_4 \\
\end{align*}

\begin{align*}
&k_1 \\
&k_3 \\
&k_2 \\
&k_4 \\
\end{align*}

\begin{align*}
&k_1 \\
&k_3 \\
&k_2 \\
&k_4 \\
\end{align*}

\begin{align*}
&k_1 \\
&k_3 \\
&k_2 \\
&k_4 \\
\end{align*}
To make better use of the conclusion, we will convert the general theorem to an equivalent but more explicit one, i.e., in the form of

\[ Y(s) = \frac{\alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + 1}{\beta_4 s^4 + \beta_3 s^3 + \beta_2 s^2 + \beta_1 s}, \]

and the conclusions are in terms of \( \alpha_i, \beta_i \). Hence the following relations are needed.

\[
\begin{align*}
\alpha_3 &= R_2 R_3 - R_6^2, \quad &\alpha_2 &= R_3, \quad &\alpha_1 &= R_2, \quad &\beta_4 &= \text{det}(R), \\
\beta_3 &= R_1 R_3 - R_5^2, \quad &\beta_2 &= R_1 R_2 - R_4^2, \quad &\beta_1 &= R_1.
\end{align*}
\]
Lemma 1

The matrix $R$ defined as

$$R := \begin{bmatrix} R_1 & R_4 & R_5 \\ R_4 & R_2 & R_6 \\ R_5 & R_6 & R_3 \end{bmatrix}$$

with entries satisfying

$$\alpha_3 = R_2 R_3 - R_6^2, \quad \alpha_2 = R_3, \quad \alpha_1 = R_2, \quad \beta_4 = \det(R),$$

$$\beta_3 = R_1 R_3 - R_5^2, \quad \beta_2 = R_1 R_2 - R_4^2, \quad \beta_1 = R_1.$$

is non-negative definite if and only if $\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3, \beta_4 \geq 0$. 
Lemma 2

The form

\[
Y(s) = \frac{(R_2 R_3 - R_6^2)s^3 + R_3 s^2 + R_2 s + 1}{s \left( \det Rs^3 + (R_1 R_3 - R_5^2)s^2 + (R_1 R_2 - R_4^2)s + R_1 \right)}
\]

with \( R \) non-negative definite and

\[
Y(s) = \frac{\alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + 1}{\beta_4 s^4 + \beta_3 s^3 + \beta_2 s^2 + \beta_1 s},
\]

with \( \alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3, \beta_4 \geq 0 \) can be expressed by each other with the corresponding coefficients equal if and only if

\[
\begin{align*}
\alpha_3 & \leq \alpha_1 \alpha_2, \\
\beta_3 & \leq \alpha_2 \beta_1, \\
\beta_2 & \leq \alpha_1 \beta_1, \\
(\beta_4 + 2\alpha_1 \alpha_2 \beta_1 - \beta_1 \alpha_3 - \alpha_1 \beta_3 - \alpha_2 \beta_2)^2 & = 4(\alpha_1 \beta_1 - \beta_2)(\alpha_2 \beta_1 - \beta_3)(\alpha_1 \alpha_2 - \alpha_3).
\end{align*}
\]
Lemma 3

Consider a non-negative definite matrix $R$ in the form of (2), and the variables $\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3,$ and $\beta_4$ satisfy

$$\alpha_3 = R_2 R_3 - R_6^2, \quad \alpha_2 = R_3, \quad \alpha_1 = R_2, \quad \beta_4 = \det(R),$$

$$\beta_3 = R_1 R_3 - R_5^2, \quad \beta_2 = R_1 R_2 - R_4^2, \quad \beta_1 = R_1.$$ (3)

Let $W := 2\alpha_1 \alpha_2 \beta_1 + \beta_4 - \alpha_1 \beta_3 - \alpha_2 \beta_2 - \alpha_3 \beta_1$, $W_1 := \alpha_1 \alpha_2 - \alpha_3$, $W_2 := \alpha_2 \beta_1 - \beta_3$, and $W_3 := \alpha_1 \beta_1 - \beta_2$. Then there exists at least one of the first- or second-order minors of $R$ being zero if and only if at least one of the variables $\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3, W_1, W_2, W_3, \beta_1 - W/(2W_1), \alpha_1 - W/(2W_2)$ and $\alpha_2 - W/(2W_3)$ is zero.
Lemma 4

Consider a non-negative definite matrix $R$ as defined in (2), whose all first- or second-order minors are non-zero, and the variables $\alpha_1$, $\alpha_2$, $\alpha_3$, $\beta_1$, $\beta_2$, $\beta_3$, and $\beta_4$ satisfy (3). Let $W := 2\alpha_1\alpha_2\beta_1 + \beta_4 - \alpha_1\beta_3 - \alpha_2\beta_2 - \alpha_3\beta_1$, $W_1 := \alpha_1\alpha_2 - \alpha_3$, $W_2 := \alpha_2\beta_1 - \beta_3$, and $W_3 := \alpha_1\beta_1 - \beta_2$. Then there exists an invertible $D = \text{diag}\{1, x, y\}$ such that $DRD$ is a paramount matrix if and only if one of the following conditions holds:

1. $W < 0$;
2. $W > 0$, $\beta_1 > (W/(2W_1))$, $\alpha_1 > (W/(2W_2))$, $\alpha_2 > (W/(2W_3))$;
3. $W > 0$, $\alpha_2 < (W/(2W_3))$ and $\beta_4 + \alpha_1\beta_3 + \alpha_3\beta_1 - \alpha_2\beta_2 \geq 0$;
4. $W > 0$, $\alpha_1 < (W/(2W_2))$ and $\beta_4 + \alpha_2\beta_2 + \alpha_3\beta_1 - \alpha_1\beta_3 \geq 0$;
5. $W > 0$, $\beta_1 < (W/(2W_1))$ and $\beta_4 + \alpha_1\beta_3 + \alpha_2\beta_2 - \alpha_3\beta_1 \geq 0$. 
Theorem 1

A positive-real function \( Y(s) \) can be realized as the driving-point admittance of a network with one inerter, one damper, and arbitrary number of springs with the assumption that \( X \) contains a well-defined impedance, if and only if \( Y(s) \) can be written in the form of

\[
Y(s) = \frac{\alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + 1}{\beta_4 s^4 + \beta_3 s^3 + \beta_2 s^2 + \beta_1 s},
\]

where the coefficients \( \alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3, \beta_4 \geq 0 \), and further satisfy the conditions of Lemma 2, and the conditions of either Lemma 3 or Lemma 4.
The Possible Cases

To express any positive-real function

\[ Y(s) = k \frac{a_0 s^2 + a_1 s + 1}{s(d_0 s^2 + d_1 s + 1)}, \]

where \( a_0, a_1, d_0, d_1 \geq 0, k > 0, \) and \( R_k \neq 0 \) in the form of

\[ Y(s) = \frac{\alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + 1}{\beta_4 s^4 + \beta_3 s^3 + \beta_2 s^2 + \beta_1 s}, \]

there are only two possible cases.
The First Case

For the first case,

\[ Y(s) = k \frac{a_0 s^2 + a_1 s + 1}{s(d_0 s^2 + d_1 s + 1)} = \frac{\alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + 1}{\beta_4 s^4 + \beta_3 s^3 + \beta_2 s^2 + \beta_1 s}, \]

the coefficients can be regarded as follows:

\[ \alpha_3 = 0, \quad \alpha_2 = a_0, \quad \alpha_1 = a_1, \quad \beta_4 = 0, \quad \beta_3 = \frac{d_0}{k}, \quad \beta_2 = \frac{d_1}{k}, \quad \beta_1 = \frac{1}{k}, \]

which are all non-negative. It’s obvious that the conditions of Lemma 3 always hold. After calculations, the conditions of Lemma 2 can be equivalent to

\[ a_0 d_1 = a_1 d_0. \]
The Second Case

For the second case, by

\[ Y(s) = k \frac{(Ts + 1)(a_0 s^2 + a_1 s + 1)}{s(Ts + 1)(d_0 s^2 + d_1 s + 1)} = \frac{\alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + 1}{\beta_4 s^4 + \beta_3 s^3 + \beta_2 s^2 + \beta_1 s}, \]

the coefficients can be regarded as follows:

\[ \alpha_3 = a_0 T, \quad \alpha_2 = a_0 + a_1 T, \quad \alpha_1 = a_1 + T, \quad \beta_4 = d_0 T/k, \]

\[ \beta_3 = (d_0 + d_1 T)/k, \quad \beta_2 = (d_1 + T)/k, \quad \beta_1 = 1/k, \quad T > 0. \]

Substitute the above coefficients into the conditions of Lemma 2, Lemma 3, and Lemma 4. After a series of calculations, the conditions can be equivalent to

\[ \frac{d_0^2}{(a_0 d_1 - a_1 d_0)(a_1 - d_1)} \geq 1, \]

and \( T \) must be expressed as

\[ T = \sqrt{\frac{a_0 d_1 - a_1 d_0}{a_1 - d_1}}. \] (4)
The Realizability Condition

**Theorem 2**

Consider a positive-real function

\[ Y(s) = k \frac{a_0 s^2 + a_1 s + 1}{s (d_0 s^2 + d_1 s + 1)}, \]

where \( a_0, a_1, d_0, d_1 \geq 0, k > 0 \) and \( R_k \neq 0 \). It can be realized as the driving-point admittance of a network with one damper, one inerter, and arbitrary number of springs but no levers with assumption that \( X \) has a well-defined impedance, if and only if

\[ \frac{d_0^2}{(a_0 d_1 - a_1 d_0)(a_1 - d_1)} \geq 1 \]

or

\[ a_0 d_1 - a_1 d_0 = 0. \]
When \( a_0 d_1 - a_1 d_0 = 0 \), it can be indicated that \( a_0 > d_0 \) since \( R_k \neq 0 \) (\( a_0 \geq d_0 \) for positive-realness).
If \( a_0 d_1 = a_1 d_0 \) and \( a_0 > d_0 = 0 \), then \( d_1 = 0 \). Therefore,

\[
Y(s) = k a_0 s + k a_1 + \frac{k}{s},
\]

with \( k_1 = k > 0 \), \( b = k a_0 > 0 \), and \( c = k a_1 \geq 0 \).
If \( a_0 d_1 = a_1 d_0 \) and \( a_0 > d_0 > 0 \), then we have

\[
Y(s) = \frac{k}{s} + k \frac{(a_0 - d_0)s + (a_1 - d_1)}{d_0 s^2 + d_1 s + 1} \\
= \frac{k}{s} + \frac{1}{\frac{d_0}{k(a_0 - d_0)} s + \frac{1}{k(a_0 - d_0)s + k(a_1 - d_1)}},
\]

with \( k_1 = k > 0 \), \( k_2 = k(a_0 - d_0)/d_0 > 0 \), \( b = k(a_0 - d_0) > 0 \), and \( c = k(a_1 - d_1) \geq 0 \).
When
\[ \frac{d_0^2}{(a_0 d_1 - a_1 d_0)(a_1 - d_1)} \geq 1, \]

\( Y(s) \) can be realized by the network as follows.
The values can be expressed as

\[ k_1 = \frac{ka_0 d_1 (a_0 d_1 - a_1 d_0) [(a_1 - d_1) T + (a_0 - d_0)]}{d_0 (a_1 - d_1) (d_1 T + d_0) [(a_0 - d_0) T + (a_0 d_1 - a_1 d_0)]}, \]

\[ k_2 = \frac{ka_0 a_1 d_1 T [(a_1 - d_1) T + (a_0 - d_0)]^2}{(a_1 - d_1) (d_1 T + d_0) [(a_0 - d_0) T + (a_0 d_1 - a_1 d_0)]^2}, \]

\[ k_3 = \frac{ka_0 T [(a_1 - d_1) T + (a_0 - d_0)]}{(d_1 T + d_0) [(a_0 - d_0) T + (a_0 d_1 - a_1 d_0)]}, \]

\[ k_4 = \frac{ka_0 T [d_0 T + (a_1 d_0 - a_0 d_1)] [(a_1 - d_1) T + (a_0 - d_0)]}{(d_1 T + d_0) [(a_0 - d_0) T + (a_0 d_1 - a_1 d_0)]^2}, \]

\[ b = \frac{ka_0^2 (a_0 d_1 - a_1 d_0) [(a_1 - d_1) T + (a_0 - d_0)]}{(a_1 - d_1) [(a_0 - d_0) T + (a_0 d_1 - a_1 d_0)]^2}, \]

\[ c = \frac{ka_0^2 d_1^2 (a_0 d_1 - a_1 d_0) [(a_1 - d_1) T + (a_0 - d_0)]^2}{(a_1 - d_1)^2 (d_1 T + d_0)^2 [(a_0 - d_0) T + (a_0 d_1 - a_1 d_0)]^2}, \]

where \( T \) is defined in (4). It is noted that \( k_1, k_2, k_3, b, c > 0 \), and \( k_4 \geq 0 \).
Conclusion

1. Introduction to passive network synthesis.

2. Introduction to the inerter.

3. An equivalent and more explicit condition for any positive-real admittance to be realizable with one damper, one inerter, and an arbitrary number of springs is established.

4. Making use the equivalent condition, a necessary and sufficient condition for the realizability of the class of positive-real functions under investigation is derived.

5. Canonical configurations to cover the condition are presented.
The End

Thank You Very Much!