1 A summary of different kinds of options

1.1 European options

The simplest financial option, a **European call option**, is the right to buy a particular asset for an agreed amount at a specified time in the future.

The prices of a European options, including call and put, been studed in the last lecture, can be computed by Black-Scholes formula.

1.2 American options

American options are contracts that may be exercised early, prior to expiry. If the underlying pays no dividend, then the price of an American call is the same as a European call. But the price of an American put is higher than a European put.

Pricing American put is still an active research topic. Three of the most recent publications are:

1. Bunch, D. S., Johnson, H., 2000, "The American Put Option and Its Critical Stock Prices," *Journal of Finance*, **55**(5), 2333-2356.

2. Carr, P., 1998, "Randomization and the American Put," *Review of Financial Studies*, **11**(3), 597-626.

3. Ju, N., 1998, "Pricing an American Option by Approximating Its Early Exercise Boundary as a Multipiece Exponential Function," *Review of Financial Studies*, **11**(3), 627-646.

1.2.1 Bermudan options

A **Bermudan option** is a type of nonstandard American option with early exercise restricted to certain dates during the life of the option. An example of a Bermudan option would be an American swap option that can be exercised only on the dates when swap payments are exchanged. See Hull (2000) pp 459.

1.3 Binary options

An **exotic option** is an option that is not a vanilla put or call.

The simplest exotic options, **Binary options**, are options that have payoffs with a certain amount of cash or nothing, which is contingent on the underlying asset value. Binary options is also called **digital options** or **cash-or-nothing options**.

The price of European Binary options have been studied in Problem Set III.

1.3.1 One-touch options

The **One-touch option** is an American version of Binary option. It can be priced with a closed form formula, see e.g., Wilmott (1998) pp 133.

1.4 Barrier options

Barrier options differ from vanilla options in that part of the option contract is triggered if the asset price hits some barrier, $S = X$, say, at any time prior to expiry.

A **down-and-out Barrier option** is the option expires worthless if the barrier $S = X$ is reached from above before expiry. Similarly we can define **up-and-in**, **down-and-in**, **up-and-out** barrier options.

Barrier options can be priced with closed form formulas, see e.g., Wilmott (1998) pp 191.

1.5 Asian options

Asian option is a kind of path-dependent option in which payoff function depends on the historical average of the underlying asset. It is called average rate option if the average price is in the position of asset price in the payoff function. It is called average strike option if the average price is in the position of strike price in the payoff function. The Asian option is separated into two types: Arithmetic Asian option and Geometric Asian option based on the way of taking averages. Geometric Asian option is easy to price and hedge since a closed-form solution is available, while Arithmetic Asian option is difficult to price and hedge.

Pricing Asian option is an active research topic. The difficulty comes from the understanding of the sum/integral of geometric Brownian motion. No closed-form solution has been found so far. But a few analytical approximations are available in the literature, including the following papers:

1. Zhang, J. E., 2003, Pricing Continuously Sampled Asian Options with Perturbation Method, *Journal of Futures Markets*, (forthcoming).

2. Zhang, J. E., 2001, A Semi-analytical Method for Pricing and Hedging Continuously Sampled Arithmetic Average Rate Options, *Journal of Computational Finance*, **5**(1), 59-79.

1.6 Lookback options

A **Lookback option** is an option that pays off some function of the realized maximum and/or minimum of the underlying asset over some prescribed period.

We can price these contracts in the Black-Scholes environment quite easily, theoretically. See, e.g., Wilmott (1998) pp 227, or read the original papers:

1. Goldman, M. B., Sosin, H. B., and Gatto, M. A., 1979, "Path Dependent Options: 'Buy at the low, Sell at the High'," *Journal of Finance*, **34**, 1111-1127.

2. Conze, A., and Viswanathan, 1991, "Path Dependent Options: The Case of Lookback Options," *Journal of Finance*, **46**, 1893-1907.

1.6.1 Russian options

A Russian option is a perpetual American Lookback option, which, at any time chosen by the holder, pays out the maximum realised asset price up to that date. It can be priced with closed form formulas, see, e.g., Wilmott, Howison and Dewynne (1995) pp 248.

1.7 Compound options

A **Compound option** is an option on an option, which gives its owner the right to purchase, for an amount E_1 at time T_1, a call with exercise price E_2 at time T_2. Its payoff function is

$$C_{\text{compound}}(S, T_1) = \max(C(S, T_1) - E_1, 0),$$
$$C(S, T_2) = \max(S - E_2, 0). \qquad T_2 > T_1$$

The closed form pricing formula can be found in the following paper:

1. Geske, R., 1979, "The Valuation of Compound Options," *Journal of Financial Economics*, **7**, 63-81.

1.7.1 As-you-like-it options

A regular **As-you-like-it option** or **Chooser option** gives its owner the right to purchase, for an amount E_1 at time T_1, *either* a call *or* a put with exercise price E_2 at time T_2. It is a "call on a call or put." Its payoff function is

$$C_{\text{As-you-like-it}}(S, T_1) = \max(C(S, T_1) - E_1, P(S, T_1) - E_1, 0)$$
$$C(S, T_2) = \max(S - E_2, 0). \qquad T_2 > T_1$$
$$P(S, T_2) = \max(E_2 - S, 0). \qquad T_2 > T_1$$

The pricing formula can be found in Wilmott (1998) pp 183.

1.8 Rainbow options

A **Rainbow option** is an option on the minimum or the maximum of several assets. Its payoff function is

$$C(S_1, S_2, \cdots, S_n, T) = \max\left(\min_{i=1}^{n} S_i - E, 0\right) \qquad (1)$$

Its price formula are given by following papers:

1. Stulz, R. M., 1982, "Options on the Minimum or the Maximum of Two Risky Assets — Analysis and Applications," *Journal of Financial Economics*, **10**, 161-185.

2. Johnson, H., 1987, "Options on the Maximum or the minimum of Several Assets," *Journal of Financial and Quantitative Analysis*, **22**(3), 277-283.

1.8.1 Exchange options

An **Exchange option** gives its owner the right to exchange one asset for another one. Its payoff function is

$$C(S_1, S_2, T) = \max(S_2 - S_1, 0). \tag{2}$$

Since $\max(S_2 - S_1, 0) = S_2 - \min(S_1, S_2)$, the pricing formula can be derived from that of Rainbow option. You may also find the formula from the original paper:

1. Margrabe, W., 1978, "The Value of an Option to Exchange One Asset for Another," *Journal of Finance*, **33**(1), 177-186.

1.9 Asian Rainbow options

An **Asian Rainbow option** is an option on the minimum or the maximum of several average prices. Its payoff function is

$$C(\bar{S}_1, \bar{S}_2, \cdots, \bar{S}_n, T) = \max\left(\min_{i=1}^{n} \bar{S}_i - E, 0\right) \tag{3}$$

Its price is studied by following paper:

1. Wu, X., and Zhang, J. E., 1999, Options on the Minimum or the Maximum of Two Average Prices, *Review of Derivatives Research*, 3(2), 183-204.

1.10 Basket options

A **Basket option** is an option written on a portfolio or basket. A typical example would be an index option.

Basket options are often approximately valued by assuming that the value of the portfolio of assets comprising the basket is lognormal rather than that each of the assets taken individually is lognormal.

1.11 Quanto options

A **Quanto option** is almost the same as regular option except the strike is measured in a different currency from the underlying. For example, a possible payoff function would be

$$C(S, T) \text{ USD} = \max(S \text{ USD} - E \text{ JPY}, 0). \tag{4}$$

The pricing formula can be found in Wilmott (1998) pp 155.

1.12 Forward-start options

A **Forward-start option** is an option that comes into being some time in the future. For example, a forward-start call option is bought now, at time $t = 0$, but with a strike price that is not known until time T_1, when the strike is set at the asset price on that date, say.

The pricing formula can be found in Wilmott (1998) pp 235.

1.13 Shout options

A **Shout call** option is a vanilla call option but with the extra feature that the holder can at any time reset the strike price of the option to the current level of the asset (if it is higher than the original strike). There is simultaneously a payment, usually of the difference between the old and the new strike prices. The action of resetting is called "shouting."

1.14 Capped-lookback and Capped-Asian options

In **Capped-lookbacks and Capped-Asians** there is some limit or guarantee placed on the size of the maximum, minimum or the average. A typical example of a capped-Asian would have the path-dependent quantity being the average of the lesser of the underlying asset and some other prescribed level. This is represented by

$$A = \frac{I}{t}, \qquad I = \int_0^t \min(S_\tau, S_{\text{cap}}) d\tau. \tag{5}$$

1.15 Lookback-asian options

A Lookback-asian option has a payoff function depends on the maximum of the asset and the average of the asset.

$$C(S,T) = \max\left(\max\left(\max_{t\in[T_0,T]} S_t, \frac{1}{T}\int_0^T S_t dt\right) - E, 0\right) \tag{6}$$

1.16 Volatility options

A **Volatility options** is an option written on the realized historical volatility, i.e.,

$$C(S,T) = \max(\sigma_h - E, 0) \tag{7}$$

$$\sigma_h = \sqrt{\frac{1}{\delta t}\frac{1}{M-1}\sum_{j=1}^{M}\left(\ln\frac{S_{t_j}}{S_{t_{j-1}}}\right)^2} \tag{8}$$

1.17 Ladder options

The **Ladder option** is a lookback option that is discretely sampled, but this time discretely sampled in asset price rather than time.

1.18 Parisian options

Parisian options are barrier options for which the barrier feature (knock in or knock out) is only triggered after the underlying has spent a certain prescribed time beyond the barrier.

1.19 Passport options

A **Perfect-trader** or **Passport option** is a call option on the trading account, giving the holder the amount in his account at the end of the horizon if it is positive, or zero if it is negative. Its payoff is

$$\max(\pi, 0) \tag{9}$$

A trader who holds the Passport option will never lose money.

References

[1] Hull, J. C., 2000, *Options, Futures, & Other Derivatives*, 4th Edition, Prentice-Hall.

[2] Wilmott, P., 1998, *Derivatives — The Theory and Practice of Financial Engineering*, John Wiley & Sons.

[3] Wilmott, P., Howison, S. & Dewynne, J., 1995, *The Mathematics of Financial Derivatives*, Cambridge University Press.