Establishing Rainfall Depth-Duration-Frequency Relationships at Daily Raingauge Stations in Hong Kong

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Abstract: Rainfall intensity (depth)-duration-frequency (IDF/DDF) relationships provide essential information in urban stormwater drainage system design and other hydrosystem infrastructures. For catchments where drainage areas are small, rainfall DDF relationships with short duration can be established based on rainfall records from automatic raingauges. Due to the progression of technology development, widespread installation of automatic raingauges does not happen until 2~3 decades ago. Therefore, record lengths at majority of automatic raingauges are relative short and the derived rainfall DDF relationships on the basis of at-site frequency analysis are potentially subject to significant sampling error. On the other hand, many conventional raingauges exist long before automatic raingauges were used. However, daily rainfall data with long records at conventional raingauges are of limited use to establish rainfall DDF relationships in areas like Hong Kong where storm duration significantly shorter than 24-hr are needed. This study presents a practical method to derive rainfall DDF relationships with short duration at conventional raingauge locations. The core components of the method include scaling model of rainfalls of different durations, establishment of relationship between annual maximum daily rainfall and rolling-time 1440min rainfall, quantification of statistical features of estimated annual maximum 1440min rainfalls, and assessment of uncertainty of derived rainfall DDF relationships at conventional raingauges.
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Introduction

• Design of stormwater drainage systems and other hydrosystem infrastructures
  - Rainfall intensity (depth)-duration-frequency (IDF/DDF)
Rainfall Data in HK

- **Automatic stations**
  - *Rainfall with fine time resolution*
  - *Short record length (<26yr)*

- **Conventional stations**
  - *Daily rainfall (3pm-3pm)*
  - *Longer record length (up to 57yr)*
  - *How to utilize the data?*
Rainfall Scaling Model (1)

- Scaling model

\[
\frac{D_{t,T}}{D_{t_b,T}} = e^{c_0(t)} \left( \frac{t}{t_b} \right)^{c_1(t)}
\]

where:
- \( t = \) rainfall duration of interest
- \( t_b = \) selected reference rainfall duration
- \( T = \) return period
- \( c_0, c_1 = \) scaling model parameters
- \( D_{t,T} = t\text{-min}, T\text{-yr rainfall depth} \)
Rainfall Scaling Model (2)

• Scaling-invariant property

\[
\frac{D_{t,T}}{D_{t_b,T}} = \left(\frac{t}{t_b}\right)^\beta
\]

where \( \beta = \) scaling model parameter independent of rainstorm duration \( t \)
Methodological Framework

Development of rainfall scaling model

- Rainfall data at automatic raingauges
- Estimating at-site scaling model parameters
  - Regionalization of scaling model parameters
  - Correlation analysis of scaling model parameters
  - Uncertainty of representative scaling model parameters

Estimation of short-duration rainfalls at conventional raingauges

- Rainfall data at conventional raingauges
- Transformation of $D_{daily}$ to $D_{1440}$
- Identification of distribution of $D_{1440}$
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Raingauge Stations in HK (1)

- Automatic raingauges (5min) in HK
  - Total: 150
  - Selected: 68 (Record length 20~26 yr)
Mean Annual Maximum Rainfall vs. Duration

- Two slopes: 0~60min  60~1440min.
Rainfall Scaling Model for Hong Kong

- $5min < t < 60min$

\[
\frac{D_{t,T}}{D_{t_b,T}} = e^{a_0 \left( \frac{t}{t_b} \right)} a_1
\]

- $60min < t < 1440min$

\[
\frac{D_{t,T}}{D_{t_b,T}} = e^{b_0 \left( \frac{t}{t_b} \right)} b_1
\]

where $(a_0, a_1)$ and $(b_0, b_1) =$ scaling model parameters in the two duration segments
Scaling-invariant property of GEV:

- If

\[ \alpha(t) = \alpha(t'), \text{ for } t \neq t' \]

- Then

\[ \frac{D_{t,T}}{D_{t_b,T}} = \left(\frac{t}{t_b}\right)^\beta \]

where \( \beta = \) scaling model parameter independent of rainstorm duration \( t \)

\( \alpha = \) shape parameter of GEV distribution
Scaling-invariant Property of GEV at Automatic Rain gauges

- Scaling-invariant property of GEV

\[ \alpha(t) = \alpha(t'), \text{ for } t \neq t' \]

*Box plots of GEV shape parameter value by duration.*

- $5\text{min} < t < 60\text{min}$
- $60\text{min} < t < 1440\text{min}$
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Contours of Scaling Model Parameters (1)

- **Duration:** $5min < t < 60min$

Parameter $a_0$  
Parameter $a_1$
- Duration: $60 \text{ min} < t < 1440 \text{ min}$

Parameter $b_0$  

Parameter $b_1$
Regionalization of Scaling Model Parameters

- K-means cluster analysis:
  - Parameter pairwise \((a_0, a_1)\): one region (the whole HK)
  - Parameter pairwise \((b_0, b_1)\): two regions
# Correlation of Scaling Model Parameters

<table>
<thead>
<tr>
<th></th>
<th>((a_0, a_1))</th>
<th>((b_0, b_1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entire Hong Kong</td>
<td>Homog. Region 1*</td>
<td>Homog. Region 2*</td>
</tr>
<tr>
<td>(\rho)</td>
<td>0.202</td>
<td>0.032</td>
</tr>
<tr>
<td>p-value</td>
<td>0.099</td>
<td>0.826</td>
</tr>
</tbody>
</table>

where \(\rho\) = correlation coefficients
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Representative Values for Scaling Model Parameters (1)

• Representative values
  - The mean values of at-site scaling model parameters within a homogeneous region

• Uncertainty
  - Bootstrap method (5000 repetitions)
  - LMR diagram is used to determine the distributions
Representative Values for Scaling Model Parameters (2)

- **L-moment ratio diagrams of the representative scaling model parameters**
  - *Normal distribution*
Representative Values for Scaling Model Parameters (3)

- Statistical properties

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(a_0)</th>
<th>(b_0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region</td>
<td>Entire Hong Kong</td>
<td>Homog. Region 1</td>
</tr>
<tr>
<td>Mean</td>
<td>0.053</td>
<td>0.007</td>
</tr>
<tr>
<td>Stdev</td>
<td>0.002</td>
<td>0.002</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(a_1)</th>
<th>(b_1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region</td>
<td>Entire Hong Kong</td>
<td>Homog. Region 1</td>
</tr>
<tr>
<td>Mean</td>
<td>0.684</td>
<td>0.372</td>
</tr>
<tr>
<td>Stdev</td>
<td>0.003</td>
<td>0.003</td>
</tr>
</tbody>
</table>
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Raingauge Stations in HK (2)

- Conventional raingauges in HK
  - Total: 250
  - Selected: 122 (Record length 20~57 yr)

Note: starred stations are utilized for the presentation purpose
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Transformation of $D_{daily}$ to $D_{1440}$ (1)

- $D_{1440} = f D_{daily} + e$

$D_{1440}$ vs. $D_{daily}$ at selected automatic raingauges in Hong Kong.
Transformation of $D_{daily}$ to $D_{1440}$ (2)

- **Weighted least square method**
- $D_{1440} = f D_{daily} + e$

**Statistical properties of the estimated coefficients.**

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>t-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$</td>
<td>1.145</td>
<td>0.00312</td>
<td>367.10</td>
<td>0.000</td>
</tr>
<tr>
<td>$e$</td>
<td>0.000</td>
<td>28.8774</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- $f$: a constant
- $e \sim N(0, 28.88^2)$
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Identifying Distribution of $D_{1440}$ at Conventional Raingauges (1)

\[ D_{1440} = f D_{daily} + e \]

Parameters of GEV, GLO, LN3, PE3
Identifying Distribution of $D_{1440}$ at Conventional Raingauges (2)

- Fitted to different distributions
  - GEV, GLO, PE3, LN3
Identifying Distribution of $D_{1440}$ at Conventional Raingauges (3)

$\mu_{D_{1440}}, \sigma_{D_{1440}}, \gamma_{D_{1440}}, \kappa_{D_{1440}} \rightarrow \kappa_{D_{1440}, \text{derived}}$

$\rightarrow \Delta \kappa$

$\rightarrow \kappa_{D_{1440}, \text{model}}$

$\Delta \kappa = |\kappa_{D_{1440}, \text{model}} - \kappa_{D_{1440}, \text{derived}}|$

No. of conventional raingauges whose best-fit distribution model for $D_{1440}$

<table>
<thead>
<tr>
<th>GEV</th>
<th>PE3</th>
<th>LN3</th>
</tr>
</thead>
<tbody>
<tr>
<td>92</td>
<td>17</td>
<td>13</td>
</tr>
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Uncertainty of Rainfall DDF at Conventional Rain Gauges

- Scaling models are subject to uncertainties
- $\ln(D_{t,T})$: Normal

**Mean**

Station: CASTLE PEAK FARM NO.31

- 5min
- 60min
- 360min
- 1440min

**Standard Deviation**

Station: HONG KONG OBSERVATORY NO.1

- 5min
- 60min
- 360min
- 1440min
Conclusions

• Scaling-invariant property of GEV is applicable in Hong Kong in two duration segments.

• Scaling model parameters can be regionalized for enhancing estimation accuracy:

\[
\begin{align*}
5 \text{min} < t < 60 \text{min} & \quad (a_0, a_1) \text{ one region;} \\
60 \text{min} < t < 1440 \text{min} & \quad (b_0, b_1) \text{ two regions.}
\end{align*}
\]

• GEV is a suitable distribution for \( D_{1440} \) at conventional stations.

• Provision of uncertainty features of estimated rainfall DDF relationships allows more prudent engineering design.
Thank You!