Financial Leverage, Job Reallocation, and Firm Dynamics over the Business Cycle

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Abstract

This paper examines the macroeconomic effects of financial leverage and firms’ endogenous entry and exit on job reallocation over the business cycle. Financial leverage and the extensive margin are the keys to explain job reallocation at both the firm-level and the aggregate level. I build a general equilibrium industry dynamics model with endogenous entry and exit, a frictional labor market, and borrowing constraints. The model provides a novel theory that financially constrained firms adjust employment more often. I characterize an analytical solution to the wage bargaining problem between a leveraged firm and workers. Higher financial leverage allows constrained firms to bargain for lower wages, but also induces higher default risks. In the model, firms adopt (S,s) employment decision rules. Because the entry and exit firms are more likely to be borrowing constrained, a negative shock affects the inaction regions of the entry and exit firms more than that of the incumbents. In the simulated model, the extensive margin explains 36% of the job reallocation volatility, which is very close to the data and is quantitatively significant.

Keywords: Firm dynamics; Job reallocation; Financial leverage; Labor market search; Business cycle.
JEL Classifications: E32, E44, J31, J63, L11.

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1 Introduction

Financial frictions are widely understood to shape macroeconomic dynamics, especially those of the recent financial crisis. Little, however, is known about how financial market frictions affect firm dynamics over the business cycle, for example, the sizable and persistent decline in new business formation observed during the Great Recession. A few studies note that entry and exit do not significantly alter aggregate dynamics. I challenge this view using empirical evidence and a quantitative model.

This paper aims to understand how financial leverage affects firms’ entry and exit decisions, namely the extensive margin, which further explains job reallocation across firms over the business cycle. I develop a general equilibrium industry dynamics model to study the determinants of job reallocation over the business cycle. I incorporate two main ingredients to the model of Hopenhayn (1992), search friction and financial leverage. In the model, firms randomly meet with workers in a frictional labor market subject to both hiring and firing cost. And firms can issue one-period debt subject to borrowing constraints. I show that the firms adopt (S,s) employment decision rules. I provide an analytical solution to the wage bargaining problem between a leveraged firm and multiple workers along the lines of Stole and Zwiebel (1996a,b).

The equilibrium outcomes adequately capture a few patterns in firm dynamics. First, entry is procyclical and exit is countercyclical. The prices of debt reflect the default probability of a firm. The model generates countercyclical external financing costs, which are important to match entry and exit dynamics (Lee and Mukoyama, 2012). Second, the extensive margin accounts for 36% of the job reallocation variance over the business cycle in the model and for 41% of that observed in the empirical data. The contribution of the extensive margin to the aggregate volatility is surprisingly large given a small fraction of entry and exit firms in the whole economy. I argue that the model cannot match both first and second moments of job reallocation without the financial leverage and decentralized bargained wage. When we increase the firing cost, both the level and the volatility of job reallocation are higher because of smaller inaction regions and higher exit rate. The financial leverage mitigates

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1 See Jermann and Quadrini (2012); Kiyotaki and Moore (2008); Mendoza (2010); Brunnermeier and Sannikov (2014). Brunnermeier et al. (2012) and Quadrini (2011) provide detailed surveys on this topic.

2 Clementi et al. (2014) shows that entry and exit amplify and propagate the effects of aggregate shocks, while Hawkins (2011), Coles and Kelishomi (2011) and Elsby and Michaels (2013) find little evidence that entry and exit shape aggregate job creation and destruction.

3 The literature documents the importance of understanding how reallocation relates to the business cycle. See Hopenhayn and Rogerson (1993); Schuh and Triest (1998); Davis et al. (1998); Hall (1998); Mortensen and Pissarides (1999) for topics on labor reallocation. See Eisfeldt and Rampini (2006); Cooper and Schott (2013) for topics on capital reallocation.

4 This is realistic as shown in empirical data that many firms have zero employment growth rates.
the reallocation at the larger and financially unconstrained firms because they can adjust leverage rather than employment. The financial leverage balances the contributions of the intensive and extensive margins to job reallocation. I prove that small firms and financially constrained firms have smaller inaction regions. Because the entry and exit firms are more likely to be borrowing constrained, a negative shock affects the inaction regions of the entry and exit firms more than that of the incumbents. Third, financially constrained firms adjust employment more frequently (Benmelech et al., 2011). This follows directly from wage bargaining outcome. Even though higher financial leverage allows constrained firms to bargain for lower wages, small and financially constrained firms still pay a higher wage than unconstrained firms. As a result, small and financially constrained are more likely to adjust employment in the presence of a productivity shock.

The model economy is populated a continuum of firms, a representative risk-neutral household with a continuum of workers, and a continuum of risk-neutral financial intermediaries. Firms are owned by the household. The firms receives an aggregate shock and an idiosyncratic shock each period. The production function exhibits decreasing returns to scale with inputs of labor. The firm pays a fixed production cost in each period. If the continuation value, conditional on the realization of aggregate and idiosyncratic shocks, is negative, the firm endogenously exits the market. A fixed number of potential entrants randomly draw an idiosyncratic productivity from a known distribution function. If they draw a good productivity, they pay a fixed entry cost and enter the market.

The firm raises funds through one-period debt. Borrowing is constrained. The debt is a standard contract with agency problems. The firm borrows money and repays the borrowed funds plus interest in the next period. The firm defaults and exits the market if it cannot fully repay its debt after the realization of shocks. The firm cannot take on excessive debt, as default is costly. Once default occurs, lenders recover only a fraction of the firm’s profits. The firm refinances its debt as its productivity varies, but its cost of doing so changes endogenously over the business cycle. The firm faces a higher default probability if it is relatively small and if the economy is in recession. Thus, the prices of debt are lower for small firms as well as during recessions. Small firms have higher external financing costs, which make it more difficult for potential entrants to enter the market during recessions.

To hire workers, firms post vacancies in a frictional labor market. Firms randomly meet with unemployed workers and offer bargained wages. Stole and Zwiebel (1996a,b) develop the standard solution to the bargaining problem between a firm and multiple workers.5

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5This bargaining solution has been widely applied in many recent papers examining firm dynamics and labor search. Examples of applications include business cycle and firm dynamics (Elsby and Michaels, 2013; Fujita and Nakajima, 2014), international trade and labor markets (Helpman et al., 2010; Felbermayr et al., 2011; Itskhoki and Helpman, 2014), product market regulation and employment (Ebell and Haefke, 2013).
They propose a unique subgame perfect equilibrium in which the wage profile coincides with the Shapley values (Shapley, 1953). I show that the bargained wage is negatively correlated with financial leverage. Borrowing constraints limit the amount of debt that firms can carry over time and directly affect wage bargaining outcomes. In the model, hiring is risky because the expected future profits may not fully repay debt. Hiring an additional worker decreases a firm’s borrowing limit and increases a firm’s likelihood of default. Because both hiring and firing are costly, employment decisions take the form of (S,s) rules. Each firm has an inaction region. Together with financial leverage, this model illustrates interesting interactions between firm dynamics and inaction regions. Small firms, financially constrained firms and potential entrants have smaller inaction regions. Both job creation and job destruction are more volatile in these firms. Moreover, the inaction regions are larger in an economy with a higher borrowing capacity or higher financial leverage.

I then calibrate the model in the stationary equilibrium. The model adequately captures some of the empirical regularities of firm dynamics. Furthermore, I consider a model without the external financing. The firms finance the projects with internal profits. In this case, the inaction regions are smaller, resulting in a less persistent employment process. The job reallocation rate is more volatile. However, it is more costly to enter the market with only internal profits. The entry and job reallocation rates decrease.

Finally, I simulate the model with an aggregate productivity shock. Firms have to predict the entire firm size distribution to compute labor market tightness when they post vacancies. I solve the model for this environment by applying standard tools from Krusell and Smith (1998). In the simulated model, the extensive margin explains 36% of the job reallocation variance over the business cycle, while the data show that the extensive margin accounts for 41% of the variance. Further, I compare the simulated economy using different values for borrowing capacity. A higher value implies a higher borrowing limit and larger inaction regions for small firms. The extensive margin contributes to a higher percentage of job reallocation variance with a higher borrowing capacity. Last, when the firms have no access to the external financing, the explanatory power of the extensive margin declines. However, the endogenous entry and exit resulting from the fixed operating cost still explains 20% of the job reallocation variance.

I organize the remainder of this paper as follows. The next section of the introduction provides the literature review. Section 2 presents the empirical patterns of cyclical job

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2009; Felbermayr and Prat, 2011), and wage and employment dynamics (Cahuc et al., 2008; Acemoglu and Hawkins, 2014).

6Brugemann et al. (2015) note that the Stole and Zwiebel game does not support the Shapley values as a subgame perfect equilibrium. Instead, they propose the Rolodex Game and present the same solution. The application of this game to the model is not affected.
reallocation at both the extensive and intensive margins. Section 3 describes the model setup. Section 4 characterizes the bargaining solution and the employment decisions. Section 5 presents the calibration and characterization of the stationary distribution. Section 6 simulates the model with aggregate shocks and shows how financial leverage interacts with entry and exit decisions as well as with job reallocation. Section 7 concludes.

Related Literature

The main contribution of this paper is to develop a model that is theoretically tractable and empirically realistic to analyze the determinants of job reallocation at both the firm-level and the aggregate level, with an emphasis on the extensive margin and financial leverage. The paper relates to several branches of the literature.

First, this paper builds on a vast literature in industry organization on firm dynamics, size, age and growth (Lucas and Prescott, 1971; Lucas, 1978; Hopenhayn, 1992; Hopenhayn and Rogerson, 1993). In these models, they characterize the stationary firm size distribution. Cooley and Quadrini (2001), Albuquerque and Hopenhayn (2004) and Cabral and Mata (2003) study the relationships among firm size, age, and financial borrowing constraints. Veracierto (2002) is the first paper to consider both aggregate shocks and idiosyncratic shocks at the firm level. Lee and Mukoyama (2012) develop a model with exogenous countercyclical entry costs to match the stylized procyclical entry rate and stable exit rate over the business cycle. In my model, I add the financial leverage to Hopenhayn (1992) model with aggregate shocks and search frictions. Debt prices are determined in equilibrium. The default cost acts as countercyclical entry and operating costs, which are the keys to explain the cyclical behavior of entry and exit.

Second, this paper is closely related to the recent literature on the relationship between firm dynamics and labor search. Kaas and Kircher (2015) and Schaal (2015) adopt directed search with Nash bargaining whereby growth firms not only are posting more vacancies but also are more likely to fill the job with a higher wage. I use the random search framework. Stole and Zwiebel (1996a,b) and Brugemann et al. (2015) provide theoretic foundations for a wage bargaining problem between a firm and multiple workers. The results have been applied in a vast and growing literature on firm dynamics with a frictional labor market. Acemoglu and Hawkins (2014) characterize a steady-state equilibrium of bargained wage and firm size distribution. Elsby and Michaels (2013) and Fujita and Nakajima (2014) analyze wage and firm size distributions, focusing on cross-sectional dispersion and aggregate dynamics. My model introduces endogenous entry and exit and financial constraints. I solve the optimal wage contract with financial leverage between a firm and multiple workers. The
solution provides a cross-sectional implication between wage dynamics and financial constraints. Financially constrained firms are more likely to adjust employment in the presence of a productivity shock, Benmelech et al. (2011). I argue that the model without both elements is not able to simultaneously match both first and second moment of job reallocation, and is not able to explain the contribution of the extensive margin to job reallocation.

Third, this paper relates to the body of literature on the role of entry and exit in shaping aggregate dynamics. Clementi and Palazzo (2013) and Clementi et al. (2014) examine the cyclical implications of endogenous firm entry and exit decisions in response to an aggregate productivity shock. Clementi and Palazzo (2013) find that the entry and exit margins account for 20% of output growth over ten years. The extensive margin also generates persistence in aggregate time series. Clementi et al. (2014) computes standard business cycle moments and impulse response functions of shocks to entry and exit rates. They find that a shock that more directly affects young firms sharpens the missing generation effect, accounting for a significant reduction in employment. Siemer (2014) finds that the financial constraint affects employment growth in small firms to a greater extent than in large firms. A large financial shock in the model generates a long-lasting recession because of reduced and persistently low entry rates. However, Hawkins (2011), Coles and Kelishomi (2011) and Elsby and Michaels (2013) find little evidence that entry and exit rates alter the behaviors of aggregate job creation and job destruction. My paper contributes to the literature with a plausible mechanism that entry and exit shapes labor market dynamics. I document new evidence that the extensive margin is important to aggregate job reallocation. To explain this, I argue that both decentralized wage bargaining and the financial leverage, as amplification mechanisms, are necessary ingredients to generate the stylized facts.

2 Job Reallocation at the Extensive Margin: Empirical Evidence

In this section, I document some facts about firm dynamics over the business cycle, including job reallocation and entry and exit rates. First, entry is procyclical, and exit is countercyclical. Second, job reallocation is very volatile over the business cycle, almost three times as the volatility of output. Third, I provide evidence that the extensive margin accounts for 41% job reallocation variance. Last, I demonstrate an important feature of firm behavior during the Great Recession. The entry rate decreased more than 25% and has remained at a low level. The job creation rate from entrants is highly correlated with entry rates and job reallocation.
2.1 Data

I use the Business Dynamics Statistics (BDS) as the primary data source. This is a reliable source of data for the job reallocation of firms and establishments in non-agricultural sectors in the US. Unlike the data in the Current Population Survey and Establishment Survey, the BDS data include cyclical movements in the aggregate employment. These data can be used to quantify different types of firms’ and margins’ contributions to fluctuations in aggregate job reallocation.

I define the intensive and extensive margins to partition all the firms and establishments included in BDS to characterize regularities in the business cycle. The intensive margin is the job creation and job destruction from incumbents. The extensive margin refers to job creation and destruction purely from entries and exits. I use this definition to demonstrate the empirical findings in the following section.

2.2 Firm Entry and Exit

I start by examining firm entry and exit dynamics over the business cycle. Figure 2 presents the entry and exit rates of firms since 1980. The rates vary significantly over the business cycle. On average, the entry rate is higher during booms and lower during recessions. The exit rate is higher during recessions and lower during booms. While the entry rate has been fairly stable since 1990, it decreased dramatically in 2007 and has remained at a low level. Moreover, there is no clear trend that business entry has recovered since 2010.

Figure 3 shows the cyclical component of the number of firms and establishments during the period from 1980 to 2012. The HP filter is subject to some measurement error in the early 1980s. The gray bars show NBER-dated recession periods. We observe that the number of firms declines during recessions, especially during the Great Recession. The total number of firms decreased by more than 25 percent.

2.3 Job Reallocation

We define job reallocation as a sum of job destruction and job creation. Figure 4 depicts the job creation and job destruction rates since 1980. Figure 5 plots cyclical component of job creation and job destruction. Table 1 reports volatility and correlation of job reallocation to output. Job destruction is more volatile than job creation. Job reallocation rate is very volatile over the business cycle, which is 2.94 times of the volatility of aggregate output. The gross job reallocation is countercyclical. The job creation rate is procyclical, and the job destruction is countercyclical. I compute the average changes of job destruction and job
Table 1: Job creation and job destruction

<table>
<thead>
<tr>
<th></th>
<th>Volatility</th>
<th>Relative volatility</th>
<th>Correlation with output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Job creation</td>
<td>0.050</td>
<td>2.78</td>
<td>0.50</td>
</tr>
<tr>
<td>Job destruction</td>
<td>0.074</td>
<td>4.11</td>
<td>-0.53</td>
</tr>
<tr>
<td>Job reallocation</td>
<td>0.053</td>
<td>2.94</td>
<td>-0.18</td>
</tr>
</tbody>
</table>

Notes: Series are HP filtered with parameter $\lambda = 6.25$. 1980-2012 Source: Own calculations. Business Dynamics Statistics (BDS).

creation in the recession relative to the change of job destruction and job creation not in the recession. There is about 2.7 times more job destruction in the recession, and 2.7 times less job creation.

However, if I only examine the job creation and destruction from entry and exit, I find different patterns. Figure 6 plots cyclical components of job creation from entry, job destruction from exit firms, entry and exit rates. The job destruction rate from exits is nearly acyclical, while the job creation rate from entry is countercyclical before 2007. This pattern occurs because of the cleansing effects. During recessions, both labor and capital are cheaper and labor is reallocated from low productivity firms to high productivity firms as well as from old firms to young firms. However, the job creation rate among new firms decreased during the Great Recession and has remained at a low level, even after the end of the recession. I further plot the relationship between job reallocation from the extensive margin and the entry and exit rates in figure 7. I find strong positive correlations among job reallocation from entry firms, entry and exit rates, and gross job reallocation. This pattern suggests that the job reallocation at the extensive margin, from entry and exit, is important for explaining reallocation dynamics.
2.4 Decomposition of Job Reallocation

Now, I decompose job reallocation into the intensive margin and the extensive margin. Denote $LR_t$ as gross job reallocation at time $t$,

\[
LR_t = \frac{\Delta L^+_{inc,t} + \Delta L^-_{inc,t}}{L_t} + \frac{\Delta L^+_{entry,t} + \Delta L^-_{exit,t}}{L_t}
\]

where $\Delta L^+_{inc,t}$ is job creation from incumbents, $\Delta L^-_{inc,t}$ is job destruction from incumbents, $\Delta L^+_{entry}$ is job creation from entrants, $\Delta L^-_{exit,t}$ is job destructions from exit, $L_t$ is total labor supply, $N_t$ is the number of firms in the market, $N^e_{exit}$ is the number of firms exiting the market, and $N^e_{entry}$ is the number of firms entering the market.

\[
\frac{\Delta L^+_{inc,t} + \Delta L^-_{inc,t}}{L_t}
\]

represents the intensive margin of job reallocation between incumbents, where labor moves from one existing firm to another. The second term, total reallocation less the intensive margin, is defined as the extensive margin of job reallocation from entry and exit. It consists of three terms. $\frac{\Delta L^+_{exit,t}}{L_t} \frac{N^e_{exit}}{N_t}$ is the relative size of exiters compared to incumbent times the exit rate, representing total job destruction from exits. $\frac{\Delta L^+_{entry}}{L_t} \frac{N^e_{entry}}{N_t}$ is the relative size of exit firms to the size of incumbent times the exit rate. This is the total number of jobs created if the number of entrants exactly replaces the number of firms exiting the market. Thus, in this case, the total number of firms does not change. The last term, $\frac{\Delta L^+_{entry}}{N^e_{entry}} \left( \frac{N^e_{entry}}{N_t} - \frac{N^e_{exit}}{N_t} \right)$, is the job creation from the changing number of firms.

Table 2 computes the variance decomposition of job reallocation between the extensive margin and the intensive margin. The data show that the extensive accounts for 36% of gross job reallocation. The extensive margin also explains 41% of job reallocation variance.
Table 2: Decomposition of Job Reallocation

<table>
<thead>
<tr>
<th></th>
<th>Percentage of mean</th>
<th>Percentage of variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extensive margin</td>
<td>36.34%</td>
<td>40.89%</td>
</tr>
<tr>
<td>Intensive margin</td>
<td>63.66%</td>
<td>59.11%</td>
</tr>
</tbody>
</table>

Notes: Series are HP filtered with parameter $\lambda = 6.25$. 1980-2012 Source: Own calculations. Business Dynamics Statistics (BDS).

3 Model

This section presents a general equilibrium industry dynamics model. The model is based on Hopenhayn (1992) and Elsby and Michaels (2013). This model differs from Elsby and Michaels (2013) in a few respects. First, I consider the interaction between financial leverage and labor bargaining problems. A firm can raise funds via one-period debt but faces a borrowing constraint. Second, my model incorporates endogenous entry and exit, which enables us to examine both the intensive and extensive margins of job reallocation. Third, I introduce a firing cost.

3.1 Preferences and Technology

The economy is populated by a continuum of firms, a representative risk-neutral household with a continuum of workers, and a continuum of risk-neutral financial intermediaries. Firms are owned by the household. All agents discount future payoffs at rate $\beta < 1$.

There are two types of firms, incumbents and potential entrants. Firms observe both aggregate and idiosyncratic productivity shocks. Firms have a decreasing returns to scale production function with only labor input. In order to hire a worker, a firm must post a vacancy at a flow cost $c_v$. A firm fires a worker at a cost $c_f$. Firms can borrowing money from the financial intermediaries, subjected to a collateral constraint. Financial intermediaries are competitive with zero profits in the equilibrium.

A fixed number of potential entrants draw an idiosyncratic productivity from a common distribution function. Given the draw, they make optimal entry decisions. The firms can borrow money from a financial intermediary, subjected to a collateral constraint. The price of debt depends on firms’ characteristics and is determined endogenously in equilibrium.

Workers can either be employed or unemployed. An unemployed worker receives flow utility $u$ from non-market activity (“leisure”) and searches for a job. An unemployed worker meets with a firm randomly subjected to the labor market friction. An employed worker obtains a bargained wage but cannot search for a new job. Wage bargaining occurs between a firm and multiple workers as in Stole and Zwiebel (1996a,b). I will discuss wage
determination shortly.

Labor market frictions limit the rate at which jobs can be filled. The matching function is $M = M(U, V)$, where $U$ and $V$ represent the total number of vacancies and unemployed workers, respectively. I assume that the matching function exhibits constant returns to scale. The vacancies posted by firms are filled with probability

$$q = \frac{M}{V} = M \left( \frac{U}{V}, 1 \right).$$

Unemployed workers find a job at a rate

$$p = \frac{M}{V} = M \left( 1, \frac{V}{U} \right).$$

Define labor market tightness

$$\theta = \frac{V}{U}.$$

With a probability of $q < 1$, posted vacancy will be filled. With a probability of $p < 1$, a worker finds a job. The market tightness is determined endogenously in equilibrium.

### 3.2 Firms

#### 3.2.1 Incumbents

Figure 1 summarizes the timeline of the model. At the beginning of period, there is a continuum of incumbent firms. The aggregate state $z$ is known. Every period is divided into three subperiods: morning, midday and afternoon. In the morning, each incumbent sequentially observes an aggregate productivity shock $z$ and an idiosyncratic productivity shock $s$. Given the realization of idiosyncratic productivity and aggregate shocks, the firm decides to remain in or exit the market. If a firm stays in the market, it incurs a fixed cost of production $c_f$ in the afternoon. The fixed cost in each period introduces the endogenous exit of a firm. There are different ways to specify endogenous exits. We can either assume that the firm has outside options (Jovanovic, 1982; Lee and Mukoyama, 2012) or that there is a fixed cost of production (Hopenhayn, 1992).

At midday, the firm adjusts its employment $n$. A firm can fire a worker at a cost of $\kappa$. Once the worker is fired, he cannot return to the market immediately. To hire a worker, firms must post vacancies. Each vacancy costs $c_v$. Given the job finding rate, firms choose their optimal level of employment. Conditional on the employment level $n$, firms bargain with workers over the labor wage $w(z, s, n, b)$. The wage depends on aggregate productivity, idiosyncratic productivity, number of employees, and debt level.
In the afternoon, the firm produces consumption goods, makes investment decisions and chooses new level of debt. The production function exhibits decreasing returns to scale

\[ F(z, s, n) = zsf(n), \]

where \( z \) is the aggregate shock, and \( n \) is the labor input. I make the following assumptions about the production function: 1) The function \( f: \mathbb{R}_+ \to \mathbb{R}_+ \) is strictly increasing, strictly concave, and continuously differentiable; and 2) both the aggregate shock \( z \) and idiosyncratic shock \( s \) are AR(1) processes.

\[
\begin{align*}
\ln s' &= \rho_s \ln (s) + \varepsilon_s \\
\ln z' &= \rho_z \ln (z) + \varepsilon_z
\end{align*}
\]

where \( \varepsilon_s \sim N(0, \sigma^2_s) \) and \( \varepsilon_z \sim N(0, \sigma^2_z) \). Denote \( h^a(z'|z) \) and \( h^i(s'|s) \) as the conditional probability density functions of idiosyncratic productivity and aggregate shocks, respectively.

A firm finances its investment by issuing one-period debt. The firm begins the period with intertemporal liabilities \( b \). Hiring a worker is risky such that default occurs in equilibrium. The firm first needs to repay the debt carried over from the last period. If it cannot fully
repay this debt, bankruptcy occurs and lenders can recover only a fraction $\zeta$ of the firm’s cash flow$^7$

$$F(z, s, n_{-1}) - w_{-1}n_{-1},$$

where $n_{-1}$ and $w_{-1}$ represent yesterday’s employment and wage, respectively. I assume that the financial intermediary takes over the firms for an extra period of production if the firms default. There is no bargaining between workers and the financial intermediary. The cost of production to the intermediary is the total compensation to workers in the period before default, $w_{-1}n_{-1}$. Upon on default, workers only receive a flow payoff $u$. The difference, $w_{-1} - u$, is forgone as part of default costs.

After firm fully repays its debt, it chooses its dividend payout $d$, and new intertemporal debt $b'$. The firm maximizes its dividend payouts $d \geq 0$. The expected present value of the firm $V^i(z, s, n_{-1}, b)$ is

$$V^i(z, s, n_{-1}, b) = \max_{d, b', n} d + \beta \mathbb{E} \max \{ V^i(z', s', n, b'), 0 \},$$

subjected to a budget constraint

$$b + w(z, s, n, b)n + d + c_f + \frac{c_v}{q(z)}(n - n_{-1})1_{n > n_{-1}} + \kappa(n_{-1} - n)1_{n < n_{-1}} \leq F(z, s, n) + q_b(z, s, n, b')b',$$

where $q_b(z, s, n, b')$ is the price of a one-period debt. The new level of debt is constrained by

$$b' \leq \chi \{ \mathbb{E}[F(z', s', n)] - w(z, s, n, b)n \},$$

where $\chi$ represents the efficiency of the financial market, $\mathbb{E}[F(z', s', n)] - w(z, s, n, b)n$ is the financial intermediary’s expected operating profits in the event of default. By varying the value of $\chi$, we can trace out all degrees of efficiency of the financial market; $\chi = 0$ corresponds to the case in which the firms have no access to external debt, and $\chi = \infty$ corresponds to a perfect financial market.

The sequential timing of decisions for an incumbent firm does not matter.$^9$ We can also assume that all of all these firm-level decisions occur simultaneously which does not change dynamics of firm behaviors. For the economic environment described here, it is impossible to know the aggregate state-contingent prices without knowing the distribution of productivity, employment and debt across firms. In particular, because the wage is now partly determined

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7I abstract from the renegotiation process here, although renegotiation is more beneficial to the financial intermediary than liquidation. See Cooley and Quadrini (2001).

8I denote lagged values using subscript $_{-1}$ and forward values with prime $'$. See also Monacelli et al. (2011).

9See also Monacelli et al. (2011).
by the labor market equilibrium, firms have to know the labor market tightness $\theta$ to predict future wages. The labor market demand curve is the aggregate labor demand across all types of firms. The distribution of firm size across all states determines the location of the labor market demand curve. Thus, firms have to incorporate such information into their decision.

### 3.2.2 Entrants

In the morning, a total measure of potential entrants $N$ randomly draw an idiosyncratic productivity $s$ from the distribution function $\Gamma(s)$. After drawing the $s$, a potential entrant decides whether to enter the market. If it enters, it pays the entry cost $c_e$ and does not carry any initial labor and debt. A firm entering the market could finance its investment through one-period debt. The new entrant maximizes the expected value of the firm $V^i(z, s, 0, 0)$, where both the initial employment and the level of debt are zero. They choose the labor input, dividend payout and debt,

$$
V^i(z, s, 0, 0) = \max_{d, b, n} \left[ d + \beta \mathbb{E} \max \{ V^i(z', s', n, b'), 0 \} \right]
$$

s.t. $d + w(z, s, n)n + \frac{n}{q(z)}c_v + c_f = F(z, s, n) + q_b(z, s, n, b')b'$

$$
\begin{align*}
&b' \leq \chi \{ \mathbb{E}[F(z', s', n)] - w(z, s, n, b)n \} \\
&d \geq 0.
\end{align*}
$$

Tomorrow morning, entrants will face exactly the same problem as the incumbent firms. Thus, firms enter the market if and only if the discounted value of entering is higher than the entry cost,

$$
V^i(z, s, 0, 0) \geq c_e.
$$

This equation determines a threshold $s^*_e(z)$ at which only firms with idiosyncratic productivity shock $s \geq s^*_e(z)$ enter the market.

### 3.3 Workers

A worker is either employed or unemployed. An employed worker receives a flow payoff equal to the bargained wage, $w(z, s, n, b)$. He loses his job with a probability of $\lambda(z', s', n, b)$ in the next period. He obtains a value of unemployment $V^u(z')$. If he continues to work at the current firm, he receives a value of $V^e(z', s', n', b')$. Thus, the value of employment in a firm
of productivity $s$, debt level $b$, and size $n$, $V^e(z,s,n,b)$, is defined as

$$V^e(z,s,n,b) = w(z,s,n,b) + \beta \mathbb{E} [\lambda (z',s',n,b') V^u(z') + (1 - \lambda (z',s',n,b')) V^e(z',s',n',b')] .$$

(3)

An unemployed worker receives a flow payoff $u$, which represents either the unemployment benefits or the value of leisure. She finds a job with probability $p$. If a worker finds a job, she receives the value of employment $V^e(z',s',n,b')$; otherwise, she remains unemployed and obtains the value of unemployment $V^u(z')$. The value of unemployment can be written as

$$V^u(z) = u + \beta \mathbb{E} [(1 - p(z')) V^u(z') + p(z') V^e(z',s',n,b')] .$$

(4)

3.4 Wage and Bargaining Problem

Wage bargaining occurs between a firm and multiple workers. The theoretical analysis of this bargaining problem is by Stole and Zwiebel (1996a,b). Stole and Zwiebel propose an extensive-form game with a unique subgame perfect equilibrium, where wages and profits coincide with the Shapley values.\(^{10}\)

Firms and workers bargain over the marginal surplus generated by their employment relationship. The marginal surplus, which I denote $J(z,s,n,b)$, is equal to

$$J(z,s,n,b) \ = \ zsfn(n) - w(z,s,n,b) - wn(z,s,n,b)n$$

$$+ \beta \mathbb{E} V_n(z',s',n,b') - \mu \chi \mathbb{E} \frac{\partial [F(z',s',n) - w(z,s,n,b)n]}{\partial n} ,$$

where $V(z,s,n,b)$ is the value of a firm, and $\mu$ is the Lagrangian multiplier of the borrowing constraint. The first term is the worker’s marginal product of labor; the second term is the worker’s wage compensation; the third term represents the worker’s marginal impact on all existing workers; the fourth term is the worker’s contribution to the firm’s future value; and the last term, which is novel in this paper, shows the worker’s marginal effect on borrowing capacity. In the case in which the firm is not borrowing constrained, $\mu = 0$, or in which the firms have no access to external debt, $\chi = 0$, the marginal surplus is identical to that in Elsby and Michaels (2013).

Wages are the outcome of a Nash bargaining game between a firm and multiple workers\(^{10}\)Brugemann et al. (2015) note that the Stole and Zwiebel game does not support the Shapley values as a subgame perfect equilibrium. Instead, they propose a Rolodex Game. Under some mild restrictions, this game has a unique subgame perfect equilibrium that satisfies the axiomatic solution offered by Shapley. The Stole and Zwiebel game and the Rolodex Game both show that each worker and firm captures a fraction $1/(n + 1)$ of the total surplus in equilibrium.
over the marginal surplus

\[(1 - \eta) [V^e (z, s, n, b) - V^u (z)] = \eta [J (z, s, n, b) + \kappa],\]

(5)

where \( \eta \) is the bargaining power of the worker. The firing cost \( \kappa \) is added to the marginal value of the firm. When the firms bargain with the workers, the workers are able to use firing cost as a credible threat to the firms. Firms have to pay firing costs if the worker-firm relationship breaks down during bargaining.

### 3.5 Exit and Default Decisions

Every morning, the firm makes its exit and default decisions. The firm exits the market if the continuation value, conditional on its productivity and aggregate productivity, is negative. A firm defaults on its debt if it cannot fully repay its debt with its current profits. If a firm defaults, I assume that there is no renegotiation and that the firm must exit the market. The firm’s debt holders can recover only part of cash flow \( \zeta [F (z', s', n) - w (z, s, n, b)] \). The default decision is important in my model, as it generates the endogenous one-period debt price for each type of firm. It also limits the growth rate of small firms. The price of debt varies with firm size and age.

The firm exits the market if and only if the firm’s value is below 0. That is

\[ V^i (z, s, n_{-1}, b) \leq 0. \]

Therefore, the exit decision involves a reservation rule

\[ v^e (z, s, n_{-1}, b) = \begin{cases} 0 & \text{if } s \geq s^*_{\text{exit}} (z, n_{-1}, b) \\ 1 & \text{o.w.} \end{cases}, \]

where

\[ s^*_{\text{exit}} (z, n_{-1}, b) = \inf \{ s \in S : V^i (z, s, n_{-1}, b) \geq 0 \}. \]

Default occurs if and only if the current period profits are smaller than the debt repayment. Equivalently, a firm will default if its debt repayment is too large or if the realized productivity shock is too small. The default will occur if and only if the firm’s idiosyncratic shock \( s \) is smaller than a cutoff value

\[ v^d (z, s, n_{-1}, b) = \begin{cases} 0 & \text{if } s \geq s^*_{d} (z, n_{-1}, b) \\ 1 & \text{o.w.} \end{cases}, \]
where

\[ s^*_d(z, n-1, b) = \sup \{ s \in S : F(z, s, n-1) - w(z-1, s-1, n-1, b-1) n-1 - c_f \leq b \} . \]

### 3.6 Financial Intermediary

Financial intermediary lends money to firms. The market is perfect competitive in which all intermediaries earn zero profits. The price of debt depends on firms’s characteristics. The firm begins the period with intertemporal liabilities \( b \). The firm first needs to repay the debt carried over from the last period. If the financial intermediary cannot receive full payment of the debt, bankruptcy occurs and they recover only a fraction \( \zeta \) of the firm’s cash flow \( F(z, s, n-1) - w_{-1} n-1 \).

I make a few more assumptions about the financial intermediary if the firm defaults. First, the intermediary takes over the firms for an extra period of production with existing workers. Second, there is no wage bargaining between workers and the financial intermediary. The cost of production to the intermediary is the total compensation to workers in the period before default, \( w_{-1} n-1 \). Third, workers only receive a flow payoff \( u \). The difference is forgone as part of default costs. Therefore, in the equilibrium, the price of debt for each type of firm, \( q_b(z, s, n, b') \), is pinned down by the following equation

\[
q_b(z, s, n, b') b' = b' \int \int \mathbb{1} (z', s', n, b') h^a(z'|z) h^i(s'|s) dsdz + \zeta \int \int [1 - \mathbb{1} (z', s', n, b')] \max\{b, F(z', s', n) - w(z, s, n, b)n\} h^a(z'|z) h^i(s'|s) dsdz,
\]

where \( \mathbb{1} (z', s', n, b') \) is an indicator function that firm continue in the market in the following period

\[
\mathbb{1} (z', s', n, b') = [1 - v^e(z', s', n, b')] [1 - v^d(z', s', n, b')].
\]

The left-hand side of the equation (6) is the total value of debt \( b' \) that financial intermediary lends to the firm at the price of \( q_b(z, s, n, b') \). On the right-hand side, the first term is the level of debt multiplied by the expected probability that the firm stays in the market and makes a full payment. The second term represents the expected liquidation value of the firm if it exits or defaults.
3.7 Recursive Equilibrium

I consider a recursive equilibrium. A key element here is the law of the motion of aggregate state of the economy. \((z, \Omega(z, s, n_{-1}, b))\) are the aggregate state variables in my model. \(z\) is exogenously given. But \(\Omega(z, s, n_{-1}, b)\), which denotes the density of firm size distributions over aggregate productivity, idiosyncratic productivity shocks, employment, and debt, is endogenously computed from the model. To hire a worker, the firms have to know the expected labor market tightness to post the exact number of vacancies. The labor market tightness depends on total labor demand of all firms. Given the exogenous process \(z\), the only objective of the firm is to know the updated \(\Omega\), that is, firms need to predict \(\Omega(z', s', n, b') = I(z', z, \Omega(z, s, n_{-1}, b))\). For convenience, I denote \(\Omega = \Omega(z, s, n_{-1}, b)\) and \(\Omega' = \Omega(z', s', n, b')\) from now on.

Given the entry, exit, and the default decisions and the policy functions of the firms, the evolution of the state of the industry \(\Omega(z', s', n, b')\) satisfies

\[
\Omega' = \Omega(z', s', n, b') = I(z', z, \Omega) = h^a(z'|z) \int_{s} \int_{n_{-1}} \int_{b} \Omega(z, s, n_{-1}, b) \cdot (z, s, n, b) \cdot h^i(s'|s) \cdot \mathbb{1}(z', s', n, b') \cdot ds \cdot dn_{-1} \cdot db
\]

+ \(h^a(z'|z) \int_{s_{\geq s^*(z, \Omega)}} \mathbb{1}(n, b'|z, \Omega) \cdot h^i(s'|s) \cdot d\Gamma(s)\),  \(7\)

where \(\mathbb{1}(n, b'|n_{-1}, b, s, z, \Omega)\) and \(\mathbb{1}(n, b'|z, \Omega)\) are the indicator functions that firms choose employment \(n\) and debt \(b'\) given incumbents and entrants’ policy functions, respectively,

\[
\mathbb{1}(n, b'|n_{-1}, b, s, z, \Omega) = \mathbb{1}_{g^i_n(s, b, n_{-1}; z, \Omega) = n, g^i_b(s, b, n_{-1}; z, \Omega) = b'},
\]

\[
\mathbb{1}(n, b') = \mathbb{1}_{g^e_n(s; z, \Omega) = n, g^e_b(s; z, \Omega) = b'},
\]

where \(g^i_n, g^i_b\) are the policy functions of the incumbents, and \(g^e_n, g^e_b\) are the policy functions of the entrants.

The Equation (7) is the evolution of probability density of firm size distribution. On the left-hand side, \(\Omega(z', s', n, b')\) is the updated distribution of firms. On the right-hand side, \(h^a(z'|z)\) represents the transition probability of aggregate state \(z\). The first part measures the transition probability of firms that continue in the market with future characteristics \(s', n, b'\), given all possible realizations from today’s states. The second part is the total measure of new entrants with employment \(n\), debt \(b'\), and idiosyncratic productivity \(s'\).

A recursive competitive equilibrium consists of a market tightness \(\theta(z, \Omega)\), a forecast rule \(I(z', z, \Omega)\), a value function \(V^i(z, s, n_{-1}, b)\), the default decision \(v^d(z, s, n_{-1}, b)\), the exit
decision $v^e(z, s, n-1, b)$, the entry decision $s^*(z)$, the price of debt $q_b(z, s, n, b')$, the optimal decision rules $\{g_n^i(s, b, n-1; z, \Omega), g_b^i(s, b, n-1; z, \Omega), g_n^e(s; z, \Omega), g_b^e(s; z, \Omega)\}$, and the number of potential entrants $N$ such that

1. Incumbent optimization: The value function $V^i(z, s, n-1, b)$ solves the Bellman Equation (1). $v^e(z, s, n-1, b)$ and $v^d(z, s, n-1, b)$ are the exit and default rules, respectively, associated with $V^i(z, s, n-1, b)$. $g_n^i(s, b, n-1; z, \Omega)$ and $g_b^i(s, b, n-1; z, \Omega)$ are associated policy functions to $V^i(z, s, n-1, b)$.

2. Entrant optimization: The value function $V^i(z, s, 0, 0)$ solves the Bellman Equation (2). $g_n^e(s; z, \Omega)$ and $g_b^e(s; z, \Omega)$ are the associated policy functions, and firms enter the market if and only if $s \geq s^*(z)$.

3. Given the forecasting function, wage function and the firms’ optimal decision rules, $V^e(z, s, n, b)$ and $V^u(z)$ solves the Bellman Equations 3 and 4.

4. The labor market clears. $w(z, s, n, b)$ solve the wage bargaining problem 5.

5. Consistency: The forecast function $I(z', z, \Omega)$ is consistent with the actual law of motion, Equation (7), implied by the optimal decision rules.

4 Bargaining Outcome and Job Reallocation

In this section, I analyze the firms’ optimal employment dynamics resulting from bargaining with workers for any given aggregate states.

**Proposition 1** The bargained wage $w(z, s, n, b)$ solves the differential equation

$$w(z, s, n, b) = \frac{\eta}{1 - \eta \mu \chi} \left[ (zs - \mu \chi \mathbb{E}[zs]) f_n(n) - (1 - \mu \chi) w_n(z, s, n, b) n + \beta p(z) \frac{c_v}{q(z)} + \kappa (1 - \beta (1 - p(z))) \right]$$

$$+ \frac{1 - \eta}{1 - \eta \mu \chi} u.$$  

(8)

**Proof.** See the Appendix. 

The solution extends the results of Elsby and Michaels (2013). If the firm is not borrowing constrained or if the firm has no access to external debt, which means $\mu \chi = 0$, the solution is identical to the results of Elsby and Michaels (2013). First, the firing cost term is included. Second, the level of debt affects the bargained wage if the firm is borrowing constrained, i.e., $\mu > 0$, and the firms have access to external debt, i.e., $\chi > 0$. The intuition for the above
solution is straightforward. The wage is a weighted average of two terms, discounted by tightness of borrowing constraints \(1 - \eta \mu \chi\). The first term is the worker’s share of contribution to firm’s value. The second term is the worker’s outside option. As shown in Stole and Zwiebel (1996a,b) and Elsby and Michaels (2013), the worker’s contribution includes the his impact on the wages of other employees of the firm. The first part, \((zs - \mu \chi \mathbb{E}[zs]) f_n(n)\), is the marginal product of labor. The second part, \((1 - \mu \chi) w_n(z, s, n, b) n\), is the worker’s impact on the wages of other employees if he quits. The last part, \(\beta p(z) \frac{c_v}{q(z)} + \kappa (1 - \beta (1 - p(z)))\), represents the marginal cost of hiring and firing the worker. The additional part is the worker’s marginal impact on the borrowing constraint, which is novel in the model. It represents the fact that the firm can strategically use debt as a bargaining tool to reduce the bargained wage.

As in a standard search model, the wage is increasing in the worker’s bargaining power \(\eta\), the marginal product of labor, the worker’s job finding rate \(p(z)\), the marginal cost of posting a vacancy for the hiring firm \(\frac{c_v}{q(z)}\), the worker’s flow value of unemployment insurance \(u\), the firm’s firing cost \(\kappa\), and the worker’s marginal impact on other workers \(w_n(z, s, n, b)\). A higher value of each parameter stands for a higher opportunity cost between the worker and the firm, resulting in a higher bargained wage. However, if a firm carries a high level of debt, it has a higher default probability. A constrained firm offers a lower bargained wage to decrease its default probability, which is beneficial to both the firm and the worker avoiding separation. By reducing bargained wage, smaller and constrained firms can carry more debt and grow faster. The benefit to the worker is the future value of the employment within the firm once it is no longer borrowing constrained.

**Proposition 2** If the production function is of the Cobb-Douglas form

\[ f(n) = n^\alpha, \]

the bargained wage \(w(z, s, n, b)\), from the differential Equation 8, has the following solution

\[ w(z, s, n, b) = A + Bn^{\alpha-1}, \]

where

\[ A = \frac{\eta}{1 - \eta \mu \chi} \left\{ \beta p(z) \frac{c_v}{q(z)} + \kappa [1 - \beta (1 - p(z))] \right\} + \frac{1 - \eta}{1 - \eta \mu \chi} u, \]

\[ B = \frac{\eta}{1 - \eta (1 - \alpha) - \eta \mu \chi \alpha} [zs - \mu \chi \mathbb{E}(zs)]. \]

**Proof.** See the Appendix. ■
A firm’s optimal employment decision \( n(z, s, n-1, b) \) is characterized by taking the first-order condition with respect to \( n \). We obtain

\[
F_n(z, s, n) - w(z, s, n, b) - w_n(z, s, n, b) n - \frac{c_v}{q(z) 1_{n>n-1}} + \kappa 1_{n<n-1} + \beta \mathbb{E} \max \left\{ V^i_n(z', s', n, b'), 0 \right\} \\
- \mu \chi \mathbb{E} \left[ F_n(z', s', n) - w_n(z, s, n, b) n - w(z, s, n, b) \right] = 0.
\]

The indicator function reflects the asymmetric marginal costs of hiring and firing workers. No cost is incurred if the firm does not change its employment level. Asymmetric marginal costs lead to an inaction region, in which the firm freezes employment such that \( n = n-1 \).

**Proposition 3** The employment decision rule is \((S, s)\), where \([\underline{s}, \overline{s}]\) is an inaction region. That is

\[
n(z, s, n-1, b) = \begin{cases} 
\overline{n}(z, s, n-1, b) & \text{if } s > \overline{s}(z, n-1, b) \\
n-1 & \text{if } s \in [\underline{s}(z, n-1, b), \overline{s}(z, n-1, b)] \\
\underline{n}(z, s, n-1, b) & \text{if } s < \underline{s}(z, n-1, b)
\end{cases},
\]

where \( \overline{n}(z, s, n-1, b) \) and \( \underline{n}(z, s, n-1, b) \) solve the following equations

\[
F_n(z, s, n) - w(z, s, n, b) - w_n(z, s, n, b) n - \frac{c_v}{q(z)} + \beta \mathbb{E} \max \left\{ V^i_n(z', s', n, b'), 0 \right\} \\
- \mu \chi \mathbb{E} \left[ F_n(z', s', n) - w_n(z, s, n, b) n - w(z, s, n, b) \right] = 0,
\]

\[
F_n(z, s, n) - w(z, s, n, b) - w_n(z, s, n, b) n + \kappa + \beta \mathbb{E} \max \left\{ V^i_n(z', s', n, b'), 0 \right\} \\
- \mu \chi \mathbb{E} \left[ F_n(z', s', n) - w_n(z, s, n, b) n - w(z, s, n, b) \right] = 0.
\]

The optimal employment decision rule takes the form of an \([s, S]\) rule, with \([\underline{s}(z, n-1, b), \overline{s}(z, n-1, b)]\) as inaction regions. Specifically, if it receives an unfavorable idiosyncratic productivity shock, the firm will shed workers until it reaches the optimal level of employment in a separation regime. If it receives a favorable idiosyncratic productivity shock, the firm will post vacancies and hire workers until it reaches the optimal level of employment in a hiring regime.

**Proposition 4** The job creation and job destruction rates for firm type \((s, n-1, b)\) and aggregate state \(z\) are

\[
JC(z, s, n-1, b) = \begin{cases} 
\frac{\overline{n}(z, s, n-1, b)}{n-1} - 1 & \text{if } s > \overline{s}(z, n-1, b) \\
0 & \text{if } s \in [\underline{s}(z, n-1, b), \overline{s}(z, n-1, b)] \\
\frac{\underline{n}(z, s, n-1, b)}{n-1} & \text{if } s < \underline{s}(z, n-1, b)
\end{cases},
\]
where JC > 0 indicates job creation; JC < 0, indicates job destruction.

**Proposition 5** If \( \rho_s = 0 \) or \( \rho_s \) is close to 1, I can show that the range of the inaction region has the following property

\[
\frac{\overline{s}(z, n_{-1}, b)}{\underline{s}(z, n_{-1}, b)} \approx 1 + \frac{c_v}{q(z)} + \kappa
\]

\[
\approx 1 + \frac{(1 - \mu\chi) [w(z, \overline{s}, n_{-1}, b) + w_n(z, \overline{s}, n_{-1}, b) n_{-1}] - \kappa}{(1 - \mu\chi) [A + B\alpha^2 n_{-1}^{\alpha-1}] - \kappa}.
\]

Further,

\[
\frac{\partial \overline{s}(z, n_{-1}, b)}{\partial \n} < 0, \quad \frac{\partial \underline{s}(z, n_{-1}, b)}{\partial \chi} > 0.
\]

**Proof.** See the Appendix. ■

This proposition shows that the relationship between the range of inaction regions and firms’ characteristics. A smaller inaction region, where firms more frequently adjust employment following a productivity shock, implies more volatile job reallocation.

First, consider a frictionless economy with competitive labor market and no financial leverage, that is \( \chi = 0 \). If firms pay competitive wages to the worker, which means the wage \( w(z, s, n_{-1}, b) \) is constant across firms and the worker’s marginal impact on other employee \( w_n(z, s, n_{-1}, b) \) is zero, then the range of inaction regions are constant for all firms.\(^{11}\)

Second, consider an economy with competitive labor market and financial leverage, that is \( \chi > 0 \). In this economy, financially constrained firms are less likely to adjust labor than financially unconstrained firms when hit by a productivity shock. First, financially constrained firms are more like to be small firms with a higher marginal product than the market clear wage. In the model, firing is less costly to firms because they can always find a worker at the competitive wage. Since the wage in the model affects collateral constraints, when a negative productivity shock hits the economy, the large firms find it is optimal to fire workers to reduce cost of default, while smaller firms find it is optimal to maintain the employment to get maximum borrowing. Financially constrained firms have a larger inaction region, which is contrary to the empirical evidence in Benmelech et al. (2011), where they find a negative and statistically significant relationship between financially constrained firms and the total change in the firm employment.

Third, consider an economy with decentralized bargained wage and no financial leverage. In my model with the decentralized wage, the inaction regions are smaller for small firms. The intuition behind this is the small firms have a higher marginal product of labor, thus

\(^{11}\) This is also a standard result in the model with adjustment cost and competitive wage.
they pay a higher wage to the worker. Because of the fixed marginal adjustment cost, small
firms are more likely to hire or fire the worker in the presence of a shock.

Fourth, consider an economy with both decentralized bargained wage and financial leverage. The range of inaction region is larger for the firms that are financially unconstrained, \( \mu \chi = 0 \). This is because, with the borrowing constraints, the financially unconstrained firms are more likely to adjust their optimal level of debt to save the cost of firing workers and searching for new workers. Their inaction regions are larger. This is also consistent with the empirical evidence in Benmelech et al. (2011).

Last, compare two economies with different level of financial borrowing capacity, more firm are financially constrained in the economy with a lower borrowing capacity. The resulting inaction regions are larger with a higher borrowing capacity than that with a lower borrowing capacity.

5 Calibration

In this section, I characterize firms’ financial behaviors and industry dynamics. By linking firms’ financial decisions to industry dynamics and the aggregate economy, the model setup allows me to examine important empirical and theoretical patterns of industry dynamics. In particular, the model without aggregate shocks, i.e., \( z = 1 \), provides the benchmark. I describe the invariant distribution of firms and their financial structures. I parameterize the model assuming that one period is one quarter. The discount factor is 0.985 with an implied annual risk-free interest rate of 6.09%. All parameter values are summarized in Table 3.

The production function exhibits decreasing returns to scale, \( z s n^\alpha \). I set \( \alpha \) to a standard value of 0.60, with an implied labor share 0.72. The idiosyncratic productivity follows an AR(1) process,

\[
\ln s' = \rho_s \ln (s) + \varepsilon_s,
\]

where \( \varepsilon_s \sim N(0, \sigma_s^2) \). The values of the persistence are obtained from Lee and Mukoyama (2012). The variance is calibrated to match the empirical patterns of employment growth volatility. The AR(1) process is approximated by a Markov process with ten states (Tauchen, 1986). The matching function is assumed to be Cobb-Douglas \( M = \xi U^\phi V^{1-\phi} \), with matching elasticity \( \phi = 0.5 \) (Petrongolo and Pissarides, 2001) and matching efficiency \( \xi = 0.129 \) (Pissarides, 2007). The workers’ bargaining power is 0.443. The unemployment benefit \( u \) is 0.387 (Elsby and Michaels, 2013).

The liquidation value of firm \( \zeta \) in the event of default is 0.7 (Gourio, 2013). I assume that potential entrants draw their productivity from the distribution \( \Gamma (s) \sim B \exp (-s) \), where \( B \)
is a scale parameter to make the distribution sum to one. The entry cost $c_e$ is endogenously computed such that only the firms with productivity above the mean productivity enter the market in the stationary equilibrium.

There are still five parameters, \{\(c_f, \chi, \sigma_s, c_v, \kappa\}\), that must be specified. Those parameters are selected to obtain the following targets: 1) a total exit probability, including the default rate, of 5.4\% \cite{Lee and Mukoyama, 2012}; 2) a mean leverage ratio of 0.81 \cite{Chen et al., 2009}; 3) variance of the employment growth rate of 0.14; 4) a hiring cost of 14\% of the quarterly wage; 5) persistence in the employment process of 0.97. \(N\) is solved such that the total measure of firm sums to 1 in the stationary equilibrium. The size of the labor force \(L\) is chosen to match a mean unemployment rate of 6.5\%.

5.1 The Economy without Aggregate Shocks

The economy is characterized by a certain distribution of firms \(\Omega\) over all state variables, \(n, b\) and \(s\). In this section, I focus on the invariant distributions of firms. The existence and the uniqueness of the invariant distribution therefore depend on the properties of the transition matrix generated by the optimal policy functions, labor and debt, of both incumbents and entrants, which is characterized by equation (7) without aggregate shocks \(z\). The invariant distribution is the fixed point of this contraction mapping.

The Appendix describes the details of the computation of the stationary distribution. I solve the value function first and then simulate the model with 10,000 firms over 5,000 periods. I exclude the first 10\% of simulations and calculate the summary statistics from the remaining simulations. To maintain the stationary distribution from endogenous exit, I replace all exiting firms with the new entrants. The total number of firms, as well as the labor supply, remains unchanged. Table 4 compares the results of the model to the data targets. The simulated model approximates the targets. The job reallocation rate also approaches the values observed in the data.

It is critical to examine how the financial leverage affects the stationary distribution of firms in the model before I proceed to solve the model with aggregate shocks. I compare the calibrated model to the model without financial leverage. I set \(\chi = 0\). If firms have no access to the external financing, they are more likely to decrease either their employment or wage profile after experiencing a negative shock. If the firms have access to the external financing, constrained firms are also more likely to hire more workers in the presence of a positive shock. However, as I have shown in the Proposition 5, the ratio of the increased productivity for hiring workers and the decreased productivity for firing workers is larger in the economy with positive \(\chi\). This is because all firms are essentially borrowing constrained
in the economy with $\chi = 0$. The inaction region is smaller, and employment growth is less persistent. Without the financial leverage, fewer firms enter with internal financing and more firms exit if their continuation values are negative. Finally, fewer jobs are created and destroyed. The intuition is straightforward. If firms have no access to the external financing, they have less incentive to hire too many workers, even they have adequate productivity. Productive firms can only finance their project and hire workers using their current operating profits, which limits their growth opportunities.

Figure 8 illustrates the bargained wage conditional on firm size. I take the number of employees as a proxy for firm size. I plot the relationship in log values. The pink line indicates the marginal product of labor. The red dotted line indicates the wage profile if firms have no debt, which is the optimal wage in the model without the financial market. The yellow line is the wage profile with an optimal level of debt. The difference between the wage with debt and the wage without debt is plotted in blue. The optimal wage contract with a Stole-Zwiebel bargaining protocol have three features. First, small firms pay higher wages because of their higher marginal product of labor and greater growth options. Second, workers are paid less than the marginal product of labor in small firms, while they are paid more than the marginal product of labor in the large firms. The workers in the small firms have to compensate their marginal negative impacts on other workers. The marginal impacts decrease with firm size. Therefore, the small firms are more likely to offer a lower wage contract than the marginal product of labor to offset the negative effect. As a firm becomes larger, the workers gain bargaining power in the wage determination process, because the marginal worker has a negligible impact. Third, the optimal wage contract with financial leverage, where small firms are constrained, is smaller than the wage contract without financial leverage. Hiring an additional worker is risky because of the default risk of small firms. Firms gain a slightly better bargaining position to offer a lower wage today and to sustain growth. Since the workers are not allowed for the on-the-job search, in this case, it is also beneficial for workers to remain in the firm with a lower bargained wage in exchange for the continuation value of employment. For larger firms, borrowing constraints are not binding. The difference between the wage with debt and the wage without debt decreases as firm size increases.

Figure 9 plots financial leverage and labor share conditional on firm size and productivity. Financial leverage is defined as the total amount of debt divided by operating profits, output less total wage payout. The labor share is defined as the total wage payout over total output. The blue and red lines plot firms’ financial leverage conditional on firm size. The pink and yellow lines represent the total labor share of an individual firm. The figure shows that 1) small firms are constrained and have higher financial leverage and that 2) more productive firms have lower financial leverage. This pattern is very intuitive. Small and low productive
firms have smaller internal profits, and hence they are willing to borrow more debt to finance their projects. As firms become larger, they can finance their projects primarily using internal cash, profits generated less dividend payouts. Larger firms pay a lower wage to each worker, but their total wage payments are higher than those of small firms.

Figure 10 plots the size, age, exit and default distributions of firms. The top panels show the firm size and age distributions. The distribution exhibits a degree of skewness toward small and young firms. All these patterns confirm the empirical regularity of the data. The bottom panels report the exit and default densities of the model as a function of firm size and age. I have shown that the default and exit probabilities are higher for small firms. The density reported is the actual number of exits and default as a fraction of total exits and defaults within each size and age group. Small and younger firms face higher default and exit probabilities, and they are more likely to default on debt and exit the market.

Figure 11 plots the dynamics of job reallocation and the growth rate conditional on firm size and age. The top panels indicate the relationships between firm dynamics and job reallocation. Job destruction is defined as the sum of employment losses from contracting firms divided by total employment. Job creation is defined as the sum of employment added from expanding firms divided by total employment. Job creation is hump-shaped. For small and young firms, the job destruction rate is also very small. This is because small and young firms demand less labor, and this effect dominates the contractionary effects of small and young firms’ higher probabilities of exit and default. As firms grow larger, their growth options dominate the default and exit probabilities, and hence the gross job reallocation rate increases. Job destruction is negatively correlated with firm size and age. However, the bottom panel of Figure 11 provides empirical evidence of firm dynamics: small and young firms grow faster and experience higher growth volatility. The growth rate and standard deviation of growth are decreasing functions of firm size and age.\footnote{Cooley and Quadrini (2001) shows that the standard deviation of growth is a decreasing function of firm size, except for very small firms.}

To summarize, this model with frictional labor and financial markets can capture both the “age dependence” and the “size dependence” of firm dynamics (Cooley and Quadrini, 2001). The model is also able to account for most of the stylized facts of firm behaviors. I replicate and extend many of the findings in Cooley and Quadrini (2001). This step is important before examining the model properties when aggregate shocks are included.
6 Aggregate Dynamics

This section presents the cyclical behaviors of firm dynamics by adding an aggregate shock. I assume that the aggregate shock follows an AR(1) process with persistence 0.9 and standard deviation 0.09. I discretize the AR(1) process into a Markov process over the state space \([0.99, 1, 1.01]\). The challenge of including both aggregate and idiosyncratic shocks is the computational complexity of the general equilibrium. My solution is based on Krusell and Smith (1998). I solve the model by iterating between an inner loop and an outer loop until the forecast rule for the aggregate state variables is consistent with the equilibrium outcome. I approximate the state variable, the distribution of firms \(\Omega\), using the first moments of market tightness \(\theta\) and productivity \(z\). The forecast rule is

\[
\log (\theta^t) = a_0 + a_1 \log (\theta) + a_2 \log (z').
\]

Given their forecast of the level of market tightness, incumbents and entrants post vacancies to fill jobs. The computation details are summarized in the Appendix. The results of the model with aggregate shocks are presented in Table 5, 6, and 7.

Table 5 shows the cyclical behavior of entry and exit. We observe a strong pattern of procyclical behavior for entries, with a rate of 4.42% during the recession versus 6.68% during the boom. The total exit rate is countercyclical: 5.85% during the recession and 5.20% during the boom. We observe both procyclical entry and countercyclical exit in the data. The model also captures countercyclical default rates of 0.58% during the recession and 0.35% during the boom. Entry is procyclical because of bargained wages and external financial costs. Countercyclical external financing costs make entry more difficult during the recession than during the boom. The average size of entrants is larger during the recession than during the boom. The average size of entrants during the boom is 92% of their average size during the recession.

Table 6 is the main result from the simulated model on the relationship between job reallocation and the extensive margin. The table presents the decomposition of job reallocation variance between the intensive and extensive margins. The extensive margin accounts for 41% of job reallocation over the business cycle in the data, while the simulated model explains 36% of job reallocation. This result is not surprising. The model features endogenous entry and exit. In response to a negative TFP shock, the option value of waiting is higher because of a higher default risk. Potential entrants delay entering the market. Incumbents fire workers and delay hiring workers. The inaction region becomes smaller. Thus, the procyclical entry rate and countercyclical exit rate adequately capture the contribution of the extensive margin to the job reallocation variance.
Table 7 summarizes the correlation between job reallocation and total output. I also compute the correlations for different size groups. The simulated time series replicate most of their counterparts in the data. Job destruction is countercyclical, with a value of -0.43 in the data versus -0.42 in the model. While job creation is nearly acyclical for the entire sample, it is procyclical for both small and large firms. Net job reallocation, defined as job creation minus job destruction, is procyclical. Gross job reallocation, defined as job creation plus job destruction, is countercyclical. The countercyclical character of job destruction largely drives the cyclicality of both net and gross job reallocation.

To examine the effectiveness of the model, I compare the simulated model results with the model in which I vary the level of borrowing capacity. Table 8 presents the model results with varying levels of borrowing capacity. It enables me to investigate the role of the financial market and the extensive margin in shaping aggregate dynamics. The extensive margin accounts for more job reallocation if we relax the borrowing constraint. The intuition is straightforward: With a higher borrowing capacity, firms face a higher default risk and reduced costs for entrants characterized by higher productivity. With a more developed financial system and a higher borrowing limit, the market is more fluid with a higher job reallocation rate. However, the economy is also more vulnerable to financial shocks, e.g., a negative shock to the borrowing limit. A permeant shock to borrowing capacity will reduce reallocation from the extensive margin. This may explain the sizable and persistent decrease in job reallocation both during and after the Great Recession. A negative financial shock increases both default probabilities and exits. At the same time, firms are less likely to enter the market. If entrants are small, their initial impacts on aggregate dynamics are limited, but these young and survivor firms grow faster and create more jobs during and after recessions. This extensive margin amplifies the effects of aggregate dynamics.

7 Conclusion

In this paper, I propose a general equilibrium industry dynamics model with endogenous entry and exit and frictional labor and financial markets. I characterize the optimal bargained wage as a function of firm characteristics. Specifically, I prove that financial leverage negatively affects the bargained wage. I numerically solve the model, which can adequately explain several stylized facts concerning both the stationary distribution (size and age dependence) and the business cycle (cyclical behaviors of entry, exit and job reallocation). The model yields a procyclical entry rate and a countercyclical exit rate. The extensive margin helps correct misallocation among firms, a more fluid labor market may explain why misallocation is more severe in developing countries.
margin resulting from endogenous cyclical entry and exit accounts for 36% of job reallocation variance over the business cycle. The model also indicates cross-sectional implications of financial leverage and the wage contract. The bargained wage is negatively correlated with firm size and financial leverage. This is an interesting empirical question left for future research.
References


Appendix

Proof of Proposition 1

Proof. First, we have firm’s first order condition w.r.t. employment

\[ F_n (z, s, n) - w (z, s, n, b) - w_n (z, s, n, b) n - \frac{c_v}{q (z)} 1_{n>n-1} + \kappa 1_{n<n-1} + \beta \mathbb{E} \max \{ V^i_n (z', s', n, b'), 0 \} \]

\[-\mu \chi \mathbb{E} [ F_n (z', s', n) - w_n (z, s, n, b) n - w (z, s, n, b) ] = 0 \]

The decision rule is \((S, s)\), with \([\underline{n}, \bar{n}]\) as inaction region. That is

\[
n (z, s, n-1, b) = \begin{cases}  
\bar{n} (z, s, n-1, b) & \text{if } s > \bar{s} (z, n-1, b) \\
n (z, s, n-1, b) & \text{if } s \in [\underline{s} (z, n-1, b), \bar{s} (z, n-1, b)] \\
\underline{n} (z, s, n-1, b) & \text{if } s < \underline{s} (z, n-1, b)
\end{cases}
\]

Therefore, we can rewrite the marginal surplus to a firm as

\begin{align*}
J (z, s, n, b) &= z s f_n (n) - w (z, s, n, b) - w_n (z, s, n, b) n + \beta \mathbb{E} V_n (z', s', n, b') \\
&\quad - \mu \chi \mathbb{E} [ F_n (z', s', n) - w_n (z, s, n, b) n - w (z, s, n, b) ] \\
&= z s f_n (n) - w (z, s, n, b) - w_n (z, s, n, b) n - \mu \chi \mathbb{E} [ F_n (z', s', n) - w_n (z, s, n, b) n - w (z, s, n, b) ] \]
\end{align*}

\[
+ \beta \iint_{s \leq z} \int_{\pi (z, s, n-1, b)} \frac{c_v}{q (z)} dH^a (z' | z) dH^i (s' | s) - \beta \iint_{s \leq z} \int_{\pi (z, s, n-1, b)} \kappa dH^a (z' | z) dH^i (s' | s) \\
+ \beta \iint_{s \leq z} \int_{\pi (z, s, n-1, b)} J (z', s', n, b') dH^a (z' | z) dH^i (s' | s),
\]

where \( H^a (z' | z) \) and \( H^i (s' | s) \) are the conditional cumulative distribution functions of idiosyncratic productivity and aggregate shocks, respectively. In addition, we can write the value of unemployment

\[
V^u (z) = b + \beta \mathbb{E} \left[ (1 - p (z)) V^u (z') + p (z) \int_z \int_s \int_{\pi (z, s, n-1, b)} \frac{V^e (z', s', n, b')}{1 - G (s (z, n-1, b))} d\Omega (z, s, n-1, b) \right].
\]

Once unemployed worker find a job, the new job must be in a firm which is posting vacancies. And \( \Omega (z, s, n-1, b) \) is the firm size distribution. Given the Nash bargaining protocol in an expanding firm, we have

\[
V^e (z, s, n, b) - V^u (z) = \frac{\eta}{1 - \eta} [ J (z, s, n, b) + \kappa ] = \frac{\eta}{1 - \eta} \left[ \frac{c_v}{q (z)} + \kappa \right]. \tag{9}
\]

we have

\[
V^u (z) = b + \beta V^u (z') + \beta p (z) \frac{\eta}{1 - \eta} \frac{c_v}{q (z)}.
\]
The value of employment can be written as

\[ V^e(z, s, n, b) = w(z, s, n, b) + \beta E \left[ \lambda(z', s', n, b) V^u(z') + (1 - \lambda(z', s', n, b)) V^e(z', s', n', b') \right] \]

\[ = w(z, n, s, b) + \beta \int_z^\infty \int_{s(z,n-1,b)}^\infty \lambda V^u(z') + (1 - \lambda) V^e(z', s', n', b') dH^a(z'|z) dH^i(s'|s) \]

\[ + \beta \int_z^\infty \int_{s(z,n-1,b)}^\infty V^e(z', s', n', b') dH^a(z'|z) dH^i(s'|s) \]

\[ + \beta \int_z^\infty \int_{s(z,n-1,b)}^\infty V^e(z', s', n', b') dH^a(z'|z) dH^i(s'|s) , \]

where

\[ \lambda = 1 - \frac{n(z, s, n-1, b)}{n-1} . \]

If the worker is fired, he transits into unemployment and receives a payoff \( V^u(z') \); otherwise, worker continues to be employed in the firm. By firm order condition, in the firm firing

workers, we have

\[ J(z, s, n, b) + \kappa = 0 . \]

Thus, we have

\[ V^e(z, s, n, b) = w(z, s, n, b) + \beta V^u(z') \]

\[ + \beta \frac{\eta}{1 - \eta} \int_z^\infty \int_{s(z,n-1,b)}^\infty \frac{c_v}{q(z)} dH^a(z'|z) dH^i(s'|s) \]

\[ + \beta \frac{\eta}{1 - \eta} \int_z^\infty \int_{s(z,n-1,b)}^\infty J(z', s', n', b') dH^a(z'|z) dH^i(s'|s) \]

\[ + \beta \frac{\eta}{1 - \eta} \int_z^\infty \int_{s(z,n-1,b)}^\infty J(z', s', n', b') dH^a(z'|z) dH^i(s'|s) . \]
This gives

\[ V^e(z, s, n, b) - V^u(z) = w(z, s, n, b) - u \]

\[ + \beta \frac{\eta}{1 - \eta} \int \int_{\pi(z, n-1, b)}^\infty \left[ \frac{c_v}{q(z)} + \kappa \right] dH^a(z'|z) dH^i(s'|s) \]

\[ + \beta \frac{\eta}{1 - \eta} \int \int_{\pi(z, n-1, b)}^\infty \left[ J(z', s', n', b') + \kappa \right] dH^a(z'|z) dH^i(s'|s) \]

\[ + \beta \frac{\eta}{1 - \eta} \int \int_{\pi(z, n-1, b)}^\infty \left[ J(n, s', b') + \kappa \right] dH^a(z'|z) dH^i(s'|s) \]

\[ - \beta p(z) \frac{\eta}{1 - \eta} \left( \frac{c_v}{q(z)} + \kappa \right) \]

\[ = w(n, s, b) - u - \beta p(z) \frac{\eta}{1 - \eta} \left( \frac{c_v}{q(z)} + \kappa \right) \]

\[ + \beta \frac{\eta}{1 - \eta} \int \int_{\pi(z, n-1, b)}^\infty \left[ \frac{c_v}{q(z)} + \kappa \right] dH^a(z'|z) dH^i(s'|s) \]

\[ + \beta \frac{\eta}{1 - \eta} \int \int_{\pi(z, n-1, b)}^\infty \left[ J(z', s', n', b') + \kappa \right] dH^a(z'|z) dH^i(s'|s) \]

\[ = \frac{\eta}{1 - \eta} \left[ J(z, s, n, b) + \kappa \right] \] (10)

Therefore, from Equation 9 and Equation 10, we have

\[ w(z, s, n, b) - u - \beta p(z) \frac{\eta}{1 - \eta} \left( \frac{c_v}{q(z)} + \kappa \right) \]

\[ = \frac{\eta}{1 - \eta} \left\{ \begin{array}{l} zs f_n(n) - w(z, s, n, b) - w_n(z, s, n, b) n \\ -\mu \chi \mathbb{E}[zs] f_n(n) - w(n, s, b) - w_n(n, s, b) n + \kappa (1 - \beta) \end{array} \right\} \]

\[ = \frac{\eta}{1 - \eta} \left\{ (zs - \mu \chi \mathbb{E}[zs]) f_n(n) - (w(z, s, n, b) - w(z, s, n, b) n) (1 - \mu \chi) + \kappa (1 - \beta) \right\} , \]

Rearrange the equation, wage is determined by

\[ w(z, s, n, b) \]

\[ = \frac{\eta}{1 - \eta \mu \chi} \left[ (zs - \mu \chi \mathbb{E}[zs]) f_n(n) - (1 - \mu \chi) w_n(z, s, n, b) n + \beta p(z) \frac{c_v}{q(z)} + \kappa (1 - \beta (1 - p(z))) \right] \]

\[ + \frac{1 - \eta}{1 - \eta \mu \chi} u. \]
Proof of Proposition 2

Proof. (Guess and verify) Assume

\[ w(z, s, n, b) = A + B\alpha^{n-1}. \]

We have

\[ A + B\alpha^{n-1} = \eta \left[ (zs - \mu\chi\mathbb{E}[zs])\alpha^{n-1} - (1 - \mu\chi)B\alpha(\alpha - 1)\alpha^{n-1} + \beta p(z) \frac{c_v}{q(z)} + \kappa (1 - \beta (1 - p(z))) \right] \]
\[ + \frac{1 - \eta}{1 - \eta\mu\chi} u. \]

Therefore

\[ B = \frac{\eta}{1 - \eta\mu\chi} [zs - \mu\chi\mathbb{E}[zs] - (1 - \mu\chi)B(\alpha - 1)], \]
\[ A = \frac{\eta}{1 - \eta\mu\chi} \left[ \beta p(z) \frac{c_v}{q(z)} + \kappa (1 - \beta (1 - p(z))) \right] + \frac{1 - \eta}{1 - \eta\mu\chi} u. \]

Solve the above two equations, I get

\[ A = \frac{\eta}{1 - \eta(1 - \alpha) - \eta\mu\alpha} \left\{ \beta p \frac{c_v}{q(z)} + \kappa [1 - \beta (1 - p(z))] \right\} + \frac{1 - \eta}{1 - \eta\mu\chi} u, \]
\[ B = \frac{\eta}{1 - \eta(1 - \alpha) - \eta\mu\alpha} [zs - \mu\chi\mathbb{E}(zs)]. \]

\[ \Box \]

Proof of Proposition 5

Assume that \( \mathbb{E}(sz) = s\rho_s\mathbb{E}(z) \), I have

\[ (1 - \mu\chi\rho s) z\alpha^{n-1} - (1 - \mu\chi) [w(z, \bar{s}, n-1, b) + w_n(z, \bar{s}, n-1, b) n-1] \]
\[ - \frac{c_v}{q(z)} + \beta \mathbb{E} \max \left\{ V_n^i(z', s', n-1, b'), 0 \right\} = 0 \]
\[ (1 - \mu\chi\rho s) z\alpha^{n-1} - (1 - \mu\chi) [w(z, \bar{s}, n-1, b) + w_n(z, s, n-1, b) n-1] + \kappa \]
\[ + \beta \mathbb{E} \max \left\{ V_n^i(z', s', n-1, b'), 0 \right\} = 0 \]

Assume

\[ \mathbb{E}V_n^i(z', s', n-1, b') \sim s\rho_s\mathbb{E}(z', s, n-1, b') \]
and $\rho_s$ is close to 1, I have
\[
\frac{\bar{s}(z, n-1, b)}{s(z, n-1, b)} \approx \left(1 - \mu \chi\right) \left[w(z, s, n-1, b) + w_n(z, s, n-1, b) n_{-1} \right] + \frac{c_v}{q(z)} \frac{c_w}{q(z)} + \kappa
\]
\[
= 1 + \frac{c_v}{q(z)} \left[w(z, s, n-1, b) + w_n(z, s, n-1, b) n_{-1} \right] - \kappa
\]
\[
= 1 + \frac{c_v}{q(z)} + \kappa
\]
\[
\left(1 - \mu \chi\right) \left[A + B\alpha^2 n_{-1}^{\alpha-1}\right] - \kappa.
\]
Take partial derivatives, it is straightforward to show that
\[
\frac{\partial s(z, n-1, b)}{\partial s(z, n-1, b)} < 0.
\]
Next, I have
\[
(1 - \mu \chi) \left[A + B\alpha^2 n_{-1}^{\alpha-1}\right]
\]
\[
= \frac{1 - \mu \chi}{1 - \eta \mu \chi} \left\{ \eta \left\{ \eta p \frac{c_v}{q(z)} + \kappa \left[1 - \beta (1 - p(z))\right] \right\} + (1 - \eta) u \right\}
\]
\[
+ \frac{1 - \mu \chi}{1 - \eta (1 - \alpha) - \eta \mu \chi \alpha} \eta [zs - \mu \chi \mathbb{E}(zs)] \alpha^2 n_{-1}^{\alpha-1},
\]
which is an increasing function of $\mu \chi$. Therefore
\[
\frac{\partial s(z, n-1, b)}{\partial q(z, n-1, b)} > 0.
\]
Computation of the Stationary Distribution

This section presents the computational details of firm dynamics in the model with aggregate shocks. I omit the notation on $z$ since it is constant here. The computational procedure is based on value function iteration and simulations.

1. Guess initial labor market tightness $\theta = \frac{U}{V}$. This gives us job finding rate, $p$, and job filling rate, $q$.

2. Set grids on $n, b$ and $\mu$. Discretize the AR(1) process of idiosyncratic shocks with ten states Markovian process.

3. Solve the value functions and optimal policy rules.
   (a) Set any initial value for $V^i_0(s, n, b)$ and $V^e_0(s, n, b)$
   (b) Compute the bargained wage conditional on $\mu$

   $$w(s, n, b; \mu) = A + B \alpha^{\alpha - 1},$$

   where

   $$A = \frac{\eta}{1 - \eta \mu \chi} \left[ \beta \frac{c_v}{q} + \kappa (1 - \beta (1 - p)) \right] + \frac{1 - \eta - u}{1 - \eta \mu \chi},$$
   $$B = \frac{\eta}{1 - \eta (1 - \alpha) - \eta \mu \chi \alpha} (s - \mu \chi \mathbb{E}[s]).$$

   (c) Calculate $V^i(s, n, b; \mu)$ by

   $$V^i(s, n_{-1}, b; \mu) = \max_{d, b', n} d + \beta \mathbb{E} \max \{ V^i(s', n, b'), 0 \} - \mu \{ \chi [\mathbb{E}F(s', n) - w(s, n, b; \mu) n] - b' \},$$

   where

   $$b' < \chi [\mathbb{E}F(s', n) - w(s, n, b; \mu) n] \text{ if } \mu = 0,$$
   $$b' = \chi [\mathbb{E}F(s', n) - w(s, n, b; \mu) n] \text{ if } \mu > 0.$$

   Therefore, I update $V^i(s, n, b)$ with

   $$V^i(s, n, b) = \max_{\mu} V^i(s, n_{-1}, b; \mu)$$

   Then obtain the optimal policy functions $g^i_n(s, n, b)$ and $g^i_b(s, n, b)$.

   (d) Solve the exit decisions $v^e(s, k, b)$

   $$v^e(z, s, n_{-1}, b) = \begin{cases} 0 & \text{if } s \geq \inf \{ s \in S : V^i(z, s, n_{-1}, b) \geq 0 \} \\ 1 & \text{o.w.} \end{cases}$$
and default decisions, $v^d(s, n-1, b)$

$$v^d(s, n-1, b) = \begin{cases} 
0 & \text{if } \omega \geq \omega^* (s, n-1, b) \\
1 & \text{o.w.} 
\end{cases}$$

where

$$\omega^* (s, n-1, b) = \sup \{s \in S : F(s, n-1) - w(s-1, n-1, b-1) n-1 - c_f \leq b\}.$$ 

(e) Update until

$$\|V^s - V^s_0\| \leq \varepsilon$$

and let $V^{*s} = V^s$.

4. Similarly, solve the entrants’ Bellman equation

$$V^e(s) = \max_{d, b'} \beta E \left[ \max \left\{ V^i (s', n, b'), 0 \right\} \right]$$

and get the optimal policy functions $g^e_n (s)$ and $g^e_b (s)$. The entry cost $c_e$ is set such that only the entrants above the mean productivity can entry the market,

$$s^* = \text{median } (s), \ c_e = V^e(s^*).$$

5. Simulate the stationary distribution of firms with 20,000 firms and 3,000 periods. Initially, all firms are new entrants. Generate the markovian chain for each firm. If the firm defaults or exits the market, it is replaced by a new entrant with a new sequence of productivity shocks.

6. Drop the first 10% simulations and calculate the invariant distributions. Compute the total labor force

$$L = \frac{\int n(s, n, b) \Omega (ds, dn, db)}{1 - U}.$$ 

where U is unemployment rate and is fixed at the targeted level. The number of separations, $S$, equals to number of hires, $M$,

$$S = \frac{1}{L} \left\{ \int_{s^* (b, n-1)}^{g(n-1, b)} [n-1 - n(s, b)] d\Omega (s, n, b) + \int_{s^* (b, n-1)}^{n-1} \Omega (s, n, b) dH^i(s'|s) \right\}$$

$$M = \frac{1}{L} \left\{ \int_{s \geq s^* (n-1, b)} [n(s, b) - n-1] d\Omega (s, n, b) + \int_{s \geq s^*_s} [n(s)] dH^i(s'|s) \right\}$$

7. Update the labor market tightness

$$\theta' = U \frac{V}{V'} = q \frac{U}{M}.$$ 

Stop until

$$\|\theta' - \theta\| \leq \varepsilon.$$
Computation of the Model with Aggregate Shocks

The section presents the computational details of firm dynamics in the model with aggregate shocks. The computational procedure is based on value function iteration and simulations.

1. Set grids on $n$, $b$, $s$ and $z$. Discretize the AR(1) process of idiosyncratic and aggregate shocks with ten-states Markovian process.

2. Guess market tightness $\theta$ a function of $z$ and $\theta_{-1}$.

   \[
   \log(\theta') = a_0 + a_1 \log(\theta) + a_2 \log(z').
   \]

3. Solve the value functions and optimal policy rules. This is similar to calculate stationary distribution, but with one more state variable.

4. Simulate the model with 20,000 firms and 3,000 periods. Initially, all firms are new entrants. Generate the aggregate shocks and markovian chain for each firm.

5. Drop the first 10% simulations and calculate the invariant distributions. Compute the total labor demand

   \[
   L(z) = \int n(z, s, n, b) \Omega (dz, ds, dn, db).
   \]

The number of separations, $S$, and the new of new hires, $M$,

\[
S(z) = \frac{1}{L} \int_{s \geq \pi(z,n_{-1},b)} [n_{-1} - n(z, s, b)] d\Omega (z, s, n, b)
+ \frac{1}{L} \int_{s < \pi'(z,b,n_{-1})} [n_{-1} (z, s, n, b)] dH^i (s'|s)
\]

\[
M(z) = \frac{1}{L} \int_{s \geq \pi(z,n_{-1},b)} [n (z, s, b) - n_{-1}] d\Omega (z, s, n, b)
+ \frac{1}{L} \int_{s \geq \pi'(z)} [n (z, s)] dH^i (s'|s)
\]

We get dynamics of unemployment

\[
U(z') = \frac{U(z) + S(z) - M(z)}{L},
\]

and market tightness

\[
\theta(z) = \frac{U(z)}{V(z)} = q(z) \frac{U(z)}{M(z)} = q(z) \frac{U(z)}{M(z)}
\]
6. Run regressions on aggregate law of motion,

$$\log (\theta') = a_0 + a_1 \log (\theta) + a_2 \log (z') + \epsilon,$$

7. update $a_0, a_1$, and $a_2$ until converge; otherwise go back to step 2.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Source or Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>0.985</td>
<td>Annual risk-free rate</td>
</tr>
<tr>
<td>Curvature of production function</td>
<td>$\alpha$</td>
<td>0.60</td>
<td>Elsby and Michaels (2013)</td>
</tr>
<tr>
<td>Liquidation value</td>
<td>$\zeta$</td>
<td>0.70</td>
<td>Gourio (2013)</td>
</tr>
<tr>
<td>Matching elasticity</td>
<td>$\phi$</td>
<td>0.5</td>
<td>Petrongolo and Pissarides (2001)</td>
</tr>
<tr>
<td>Matching efficiency</td>
<td>$\xi$</td>
<td>0.129</td>
<td>Pissarides (2007)</td>
</tr>
<tr>
<td>Worker’s bargaining power</td>
<td>$\eta$</td>
<td>0.443</td>
<td>Elsby and Michaels (2013)</td>
</tr>
<tr>
<td>Value of unemployment</td>
<td>$u$</td>
<td>0.387</td>
<td>Elsby and Michaels (2013)</td>
</tr>
<tr>
<td>Aggregate productivity shock persistence</td>
<td>$\rho_z$</td>
<td>0.9</td>
<td>Standard in the literature</td>
</tr>
<tr>
<td>Aggregate productivity shock volatility</td>
<td>$\sigma_z$</td>
<td>0.09</td>
<td>Standard in the literature</td>
</tr>
<tr>
<td>Idiosyncratic productivity shock persistence</td>
<td>$\rho_s$</td>
<td>0.97</td>
<td>Lee and Mukoyama (2012)</td>
</tr>
<tr>
<td>Idiosyncratic productivity shock volatility</td>
<td>$\sigma_s$</td>
<td>0.1045</td>
<td>Variance of employment growth</td>
</tr>
<tr>
<td>Tightness of borrowing constraint</td>
<td>$\chi$</td>
<td>1.5</td>
<td>Leverage ratio of 0.81</td>
</tr>
<tr>
<td>Vacancy posting cost</td>
<td>$v_c$</td>
<td>0.13</td>
<td>Resource cost associated with hiring</td>
</tr>
<tr>
<td>Firing cost</td>
<td>$\kappa$</td>
<td>0.16</td>
<td>Persistent of employment</td>
</tr>
<tr>
<td>Fixed production cost</td>
<td>$c_f$</td>
<td>1.7</td>
<td>Quarterly exit rate</td>
</tr>
<tr>
<td>Entry cost</td>
<td>$c_e$</td>
<td>2.8</td>
<td>Entry cutoff</td>
</tr>
<tr>
<td>Measure of potential entrants</td>
<td>$N$</td>
<td>0.2</td>
<td>Measure of all firms</td>
</tr>
</tbody>
</table>

Table 3: Parameterizations

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</table>

Table 4: Data and model statistics in the stationary equilibrium

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Table 5: Entry and exit with aggregate shocks
Table 6: Decomposition of job reallocation variance between groups

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extensive margin</td>
<td>41%</td>
<td>36%</td>
</tr>
<tr>
<td>Intensive margin</td>
<td>59%</td>
<td>64%</td>
</tr>
<tr>
<td>Relative volatility</td>
<td>2.94</td>
<td>3.03</td>
</tr>
</tbody>
</table>

Notes: Series are HP filtered with parameter $\lambda = 6.25$. 1980-2012 Source: Own calculations. Business Dynamics Statistics (BDS).

Table 7: Correlation of job reallocation to output

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Job reallocation</td>
<td>-0.18</td>
<td>-0.22</td>
</tr>
<tr>
<td>Job creation</td>
<td>0.50</td>
<td>0.40</td>
</tr>
<tr>
<td>Job destruction</td>
<td>-0.53</td>
<td>-0.42</td>
</tr>
</tbody>
</table>

Notes: Series are logged and HP filtered with parameter $\lambda = 6.25$. 1980-2012 Source: Own calculations. Business Dynamics Statistics (BDS).

Table 8: Job reallocation with different borrowing capacity

<table>
<thead>
<tr>
<th></th>
<th>$\chi = 5$</th>
<th>$\chi = 1.5$</th>
<th>$\chi = 0.5$</th>
<th>$\chi = 0$</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extensive margin</td>
<td>0.44</td>
<td>0.36</td>
<td>0.26</td>
<td>0.20</td>
<td>0.41</td>
</tr>
<tr>
<td>Intensive margin</td>
<td>0.56</td>
<td>0.64</td>
<td>0.74</td>
<td>0.80</td>
<td>0.59</td>
</tr>
<tr>
<td>Relative volatility</td>
<td>1.66</td>
<td>3.03</td>
<td>3.25</td>
<td>4.08</td>
<td>2.94</td>
</tr>
</tbody>
</table>
Figure 2: Entry and exit rate

Notes: Series are logged and HP filtered with parameter $\lambda = 6.25$. 1980-2012 Source: Own calculations. Business Dynamics Statistics (BDS). The gray bars are NBER-dated recession periods.
Figure 4: Job creation and job destruction

Notes: Series are logged and HP filtered with parameter $\lambda = 6.25$. 1980-2012 Source: Own calculations. Business Dynamics Statistics (BDS). The gray bars are NBER-dated recession periods.
Figure 5: Cyclical component of job creation and job destruction

Notes: Series are logged and HP filtered with parameter $\lambda = 6.25$. 1980-2012 Source: Own calculations. Business Dynamics Statistics (BDS). The gray bars are NBER-dated recession periods.
Figure 6: Cyclical component of job reallocation: the extensive margin, entry and exit rates

Figure 7: Job reallocation, entry and exit rate

Source: Own calculations. Business Dynamics Statistics (BDS) 1980-2012. The gray bars are NBER-dated recession periods. The red lines are fitted value from OLS regressions.
Figure 8: Bargained wage conditional on firm size and debt
Figure 9: Financial leverage and labor share conditional on firm size and productivity
Figure 10: Firm size and age distributions
Figure 11: Job reallocation and firm growth distribution

- **Job Reallocation and Mean Growth**: The graph shows the distribution of job reallocation and mean growth for firms of different sizes and ages.

- **Standard Deviation of Growth**: The graph also illustrates the standard deviation of growth over the same time periods.

- **Job Creation and Job Destruction**: The top graphs display the job creation and destruction rates for firms of different sizes and ages.