A two-fluid model of turbulent two-phase flow for simulating turbulent stratified flows

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Received 7 July 2001; received in revised form 28 November 2001; accepted 31 January 2002

Abstract

In this paper, a two-fluid model of turbulent two-phase flow is used to simulate turbulent stratified flows. This is a unified multi-fluid model for the motion of each phase in the flow, whose turbulent transport is closed by a two-phase k-ε model. The exchanges of mass, momentum and energy between the two phases are fully accounted for in the simulation. For illustration, a case of turbulent stratified flow with strong buoyancy effects, for which extensive experimental data are available, is selected for examination. It is shown that the numerical results agree well with the experimental data. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Buoyancy effects; Stratified flow; Two-phase turbulence; Two-fluid model

1. Introduction

Complex stratified flows abound in natural and engineering processes. In problems pertinent to coastal and ocean engineering, environmental pollution, and many other fields of engineering, there often exist flows with a non-uniform distribution of density or temperature, which in some cases may even have a sharp gradient of change. Such an abrupt spatial change of a variable often makes the problem not amenable to simple mathematical analyses. Predictive turbulence models for the environmental flow and transport have recently been presented by Ni (1996) and Shen et al. (2002).
These models have made some success. However they do not account for the mixing structure of a flow field in sufficiently fine detail. A problem that is characterized by appreciable phase exchanges of mass, momentum and energy can be too complicated to be accurately simulated by a simple turbulence closure model. It is of practical importance if a model is able to distinguish the various components of a flow field in the course of simulation. Spalding (1984) proposed a two-fluid model of
turbulence (see also Markatos, 1985), which was later improved by Fan (1988). A distinct feature of the model is to treat a general phase as a substance that can be distinguished by a property parameter such as temperature, pressure, velocity, density, and so on. Based on the framework of the two-fluid model, it is the intention of this work to present a two-fluid model of turbulent two-phase flow for the simulation of turbulent stratified flows. Computational results are shown to agree well with some published experimental data.

2. Physical concepts

A central part of the two-fluid model of turbulent two-phase flow is to consider such flow being composed of the interacting turbulent motions of two fluids, which coexist in time and space but possess different volume fractions. These two fluids in turbulent motion can be regarded as interpenetrating continua, which obey their own governing differential equations. The two fluids can have exchanges of mass, momentum and energy with each other. For identification, each fluid carries a distinct phasal mark that remains invariant throughout the flow. Let \( k = 1 \) and \( k = 2 \) be used to mark the two different fluids in the flow. Then the related properties of such a two-fluid model can be expressed in the following sections.

2.1. Mixture parameters of two-phase flow

Any parameter for a mixture of two phases can be obtained by taking a weighted average of the corresponding parameters for each phase. The volume-average and mass-average are the two typical ways of deriving the parameters of the mixture, which are defined as follows:

Volume-average:

\[
R = \Phi_1 R_1 + \Phi_2 R_2
\]

Mass-average:

\[
R = (\Phi_1 \rho_1 R_1 + \Phi_2 \rho_2 R_2) / \rho
\]
\[
\rho = \Phi_1 \rho_1 + \Phi_2 \rho_2
\]

where \( \Phi_1 \) and \( \Phi_2 \) are respectively the volume fractions of the first and second phases, \( \rho_1 \) and \( \rho_2 \) are the respective substance densities of the two phases, and \( R \) is a generic variable that can be the velocity, turbulence properties, and so on.

2.2. Pressure of two-phase flow

There are two alternative ways of defining the mixture pressure of two-phase flow. By the first definition, the mixture pressure is assumed to be composed of partial pressures due to individual phases. It is further assumed that the partial pressure of a phase amounts to the total pressure multiplied by the phase volume fraction \( \Phi_k \).
This concept is parallel to that of a gas mixture. The phase pressure \( P_k \) and the total pressure \( P \) are therefore related by:

\[
P = \sum_k P_k(P_k = P \Phi_k) \tag{4}
\]

Alternatively, the same pressure is assumed for each phase and the mixture, i.e. \( P_k(k = 1, 2) = P \). The phasal pressure gradient in the momentum equation is, however, the pressure gradient weighted by the volume fraction. These two ways of defining the mixture pressure are related to each other if the contribution to the pressure gradient is expressed as:

\[
\delta_{pi} = -\alpha_p \rho_k \frac{\partial \Phi_k}{\partial X_i} - \Phi_k \frac{\partial P}{\partial X_i} \tag{5}
\]

When \( \alpha_p = 1 \), \( \delta_{pi} = -\partial (\rho \Phi_k)/\partial X_i = -\partial P_k/\partial X_i \), which is the former way. When \( \alpha_p = 0 \), then \( \delta_{pi} = -\partial \rho_k \partial P/\partial X_i \), which corresponds to the latter way. For convenience, the second way of defining the mixture pressure is adopted in this paper, i.e. \( \alpha_p = 0 \).

3. Mathematical model

On applying the Reynolds decomposition and averaging to the instantaneous Navier–Stokes equations, while modeling the correlation terms with a gradient-flux closure, we obtain the governing equations that describe the transport of averaged parameters in a turbulent two-phase flow system.

The time-averaged continuity equation (volume fraction equation) for phase \( k \) is

\[
\frac{\partial}{\partial t}(\rho_k \Phi_k) + \frac{\partial}{\partial X_j}(\rho_k \Phi_k U_{kj}) = \frac{\partial}{\partial X_j} \left( \Phi_k \frac{\mu_{ek}}{\sigma_{\Phi_k}} \frac{\partial \Phi_k}{\partial X_j} + \Phi_k \frac{\mu_{el}}{\sigma_{\Phi_l}} \frac{\partial \Phi_l}{\partial X_j} \right) + S_{k,l} + m_{k,l} \tag{6}
\]

The time-averaged momentum equation for phase \( k \) is

\[
\frac{\partial}{\partial t}(\rho_k \Phi_k U_{ki}) + \frac{\partial}{\partial X_j}(\rho_k \Phi_k U_{kj} U_{ki}) = \frac{\partial}{\partial X_j} \left[ \Phi_k \frac{\mu_{ek}}{\sigma_{\Phi_k}} \left( U_k \frac{\partial \Phi_k}{\partial X_j} + U_h \frac{\partial \Phi_h}{\partial X_j} \right) \right]
\]

\[
+ \frac{\partial}{\partial X_j} \left[ \Phi_k \frac{\mu_{el}}{\sigma_{\Phi_l}} \left( \frac{\partial U_{ki}}{\partial X_j} + \frac{\partial U_{kj}}{\partial X_i} \right) \right] - \Phi_k \frac{\partial R}{\partial X_i} + \Phi_k (\rho_k - \rho_r) g_i + F_{ki} \tag{7}
\]

The transport equation of turbulent kinetic energy for phase \( k \) is

\[
\frac{\partial}{\partial t}(\rho_k \Phi_k e_k) + \frac{\partial}{\partial X_j}(\rho_k \Phi_k U_{kj} e_k) = \frac{\partial}{\partial X_j} \left( \frac{\mu_{ek}}{\sigma_{\Phi_k}} \frac{\partial \Phi_k}{\partial X_j} \right) + \frac{\partial}{\partial X_j} \left( \frac{\mu_{ek}}{\sigma_{\Phi_k}} \frac{\partial \Phi_k}{\partial X_j} \right)
\]

\[
- \rho_k \Phi_k e_k + G_k + B_k + \alpha_f \left( \frac{\mu_{ek}}{\sigma_{U_k}} \frac{\partial U_{ki}}{\partial X_i} - \frac{\mu_{el}}{\sigma_{U_l}} \frac{\partial U_{li}}{\partial X_i} \right) \tag{8}
\]
The transport equation of turbulent kinetic energy dissipation rate for phase $k$ is

$$\frac{\partial}{\partial t}(\rho_k \Phi_k \varepsilon_k) + \frac{\partial}{\partial x_j}(\rho_k \Phi_k \varepsilon_k \dot{U}_{kj}) = \frac{\partial}{\partial x_j}\left(\frac{\mu_{ek}}{\sigma_{\varepsilon_k}} \frac{\partial \varepsilon_k}{\partial x_j}\right) + \frac{\partial}{\partial x_i} \left(\frac{\mu_{ek}}{\sigma_{\varepsilon_k}} \frac{\partial \Phi_k}{\partial x_i}\right)$$  \hspace{1cm} (9)

$$+ \frac{\varepsilon_k}{k_k} \left( C_1 G_k - C_2 \rho_k \Phi_k \varepsilon_k \right) - 2 \alpha_f \rho \varepsilon_k$$

where

$$F_{ki} = K_f \rho \Phi_k \Phi_i \left[U_{ki} - U_{il} \right] \left(U_{ki} - U_{li} \right) \varepsilon \left/ k^{1.5} \right.$$  \hspace{1cm} (10)

$$S_{k,l} = E'' = K_m \rho \Phi_k \Phi_i \left[U_{ki} - U_{li} \right] \varepsilon \left/ k^{1.5} \right.$$  \hspace{1cm} (11)

$$m_{k,l} = \left( \frac{\mu_{el}}{\sigma_{\Phi_i}} - \frac{\mu_{ek}}{\sigma_{\Phi_k}} \right) \frac{\partial \Phi_i}{\partial x_j} \frac{\partial \Phi_i}{\partial x_j}$$  \hspace{1cm} (12)

$$G_k = \frac{\partial U_{ki} \left[ \mu_{ek} \frac{\partial U_k}{\partial x_i} + \frac{\mu_{ek}}{\sigma_{\Phi_k}} \left( \frac{\partial U_{ki}}{\partial x_j} + \frac{\partial U_{kj}}{\partial x_i} \right) \right]}{\partial x_j}$$  \hspace{1cm} (13)

$$B_k = - \frac{\mu_{ek}}{\sigma_{\Phi_k}} \left( \frac{\rho_k - \rho_r}{\rho_k} \right) \frac{\partial \Phi_k}{\partial x_i} g_i$$  \hspace{1cm} (14)

$$\alpha_f = \frac{1}{\rho} K_f E''$$  \hspace{1cm} (15)

$$\mu_{ek} = \rho_k C_{\mu} \frac{k_i^2}{\varepsilon_k}$$  \hspace{1cm} (16)

$$\mu_{ek} = \mu_k + \mu_{ek}$$  \hspace{1cm} (17)

In the equations presented above, the symbols have the following meanings: $t$ is time; subscripts $k$ and $l$ are the fluid marks which may be either 1 or 2 (but $k \neq l$); $F_{ki}$ is the component of the time-averaged frictional force of phase $k$ in the $i$ direction; $U_{ki}$ and $U_{li}$ are the components of the time-averaged velocity of phases $k$ and $l$ in the $i$ direction, respectively; $\Phi_k$ and $\Phi_i$ are the time-averaged volume fractions of phases $k$ and $l$, respectively; $\rho$, $k$ and $\varepsilon$ are the density, turbulent kinetic energy and turbulent kinetic energy dissipation rate of mixture, respectively; $P$ is the time-averaged pressure of the mixture; $\mu_k$, $\mu_{ek}$ and $\mu_{ek}$ are the dynamic viscosity, dynamic eddy viscosity and effective dynamic viscosity of phase $k$, respectively; $\rho_k$ is the substance density of phase $k$; $\rho_r$ is the substance density of ambient fluid; $\sigma_{\Phi_k}$ is the turbulent Schmidt number of phase $k$ (which is constant in general, but strongly influenced by buoyancy effects that can be accounted for with the Munk–Anderson empirical formula, Rodi, 1993); $g_i$ is the component of gravitational acceleration in the $i$ direction; $K_f$, $K_m$, $\sigma_k$, $\sigma_{\varepsilon_k}$, $C_1$, $C_2$ and $C_\mu$ are all empirical constants for which the following values are chosen: $K_f = 0.05$, $K_m = 0.1$, $\sigma_k = 1.0$, $\sigma_{\varepsilon_k} = 1.3$, $C_1 = 1.44$, $C_2 = 1.92$ and $C_\mu = 0.09$. 
4. Numerical computation and discussion on results

A two-fluid model system is obviously much more complex to compute than a one-fluid system. For one thing, the number of differential equations for a two-fluid model is twice that for a one-fluid model. Also, the non-linearity and coupling of the equations are much stronger owing to interactions between the two fluids. The equations are coupled not only within those for the same phase, but also among those for different phases. On the other hand, the solution to the coupled equations is highly sensitive to the initial values. Numerical efficiency is also strongly affected by the methods for discretization and iteration, and the way of relaxation. These computational difficulties can be mitigated only through taking good care of the non-linearity and coupling. In this regard, the numerical method that approaches the two-fluid system from a no-slip one-fluid system is used.

A unified multi-fluid model is adopted here, and the two-phase flow can be computed by finite differences in a unified Eulerian field. In this work, the second-order implicit SIMPLE scheme is extended to the two-phase flow. The pressure correction equations are obtainable from the continuity equation for the mixture. By treating a two-fluid system from the approach of a no-slip one-fluid system, the two-phase coupling system is numerically solved with an under-relaxation iterative scheme.

4.1. Numerical example

The two-fluid model described above is applied to a problem as shown in Fig. 1, for which the extensive data available from Uittenbogaard (1988) can be used for comparison. In the experiment, a turbulent mixing layer was created between two streams of water, initially separated by a splitter plate, having different velocities and salinity concentrations. The two streams were maintained at a constant (and equal) temperature but with a density difference of 15 kg/m³. The denser stream was below the lighter one, a condition that gives rise to stable stratification leading to suppression of turbulent mixing. This flow problem was adopted as a test case for ‘The Fourteenth Meeting of the IAHR Working Group on Refined Flow Modelling’ (Scheuerer, 1989).

The boundary conditions at the inlet cross-section, comprising profiles of the dependent variables, were specified in the experiment, while at the exit the flow was already fully developed. The top was a slip plane (rigid-lid approximation) and the

![Fig. 1. Flow geometry and coordinate system.](image-url)
bottom was a smooth wall where the velocity follows a standard log-law and turbulence is in local equilibrium.

4.2. Computational results and discussion

Comparisons between the computed and the measured profiles of velocity $U$, turbulent kinetic energy $k$, and relative density $\rho_{rel} = (\rho - \rho_1)/(\rho_2 - \rho_1)$ for the mixture are presented in Figs. 2, 4 and 6. The computed profiles of velocity $U$ and turbulent kinetic energy $k$ for the two fluids are shown in Figs. 3 and 5. These profiles are located at 5, 10 and 40 m downstream of the splitter plate. Clearly, the predictions for the velocity and relative density are in very close agreement with the data in all stages of the flow development. It is reproduced in the computation that the initially
distinct boundary layers that were developed on either side of the splitter plate have almost merged into a single shear layer akin to a boundary layer developing over the flume’s bed. The predictions also clearly show that the turbulent kinetic energy has completely collapsed in the outer part of the shear layer under the influence of strong stable stratification.

It is also apparent from the velocity and relative density profiles that the presence of buoyancy makes the phase coupling to be of the ‘check type’, i.e. the phase coupling does not tend to make the distribution of parameters uniform but tends to make the parameters maintain their own identity. The upper lighter stream flows forward while attempting to diffuse into the lower layer. On the other hand, the lower denser stream holds out against the lighter one to restrain it from descending. The density stratification is hence developed and sustained, although the turbulent entrainment between the two phases continues to take place. The intensity of entrainment decreases with an increase in density difference between the two streams. The profiles of the averaged parameters obviously provide a comprehensive description of the complex internal process. Therefore the two-fluid model can not only simulate the phenomenal movement of a fluid, but also reveal fine details of the internal processes resulting from the complex movement of the fluid. This is in sharp contrast to a one-fluid model.
5. Summary and concluding remarks

(a) The two-fluid model of turbulent two-phase flow is capable of not only simulating the phenomenal movement of a fluid, but also revealing details of the internal processes of the mixing in a two-phase flow. One-fluid models fall short in doing so.
(b) A two-fluid model has twice the number of differential equations as that for a one-fluid model. The interactions between the two fluids lead to stronger non-linearity and the coupling of equations. To circumvent these difficulties, the numerical method that approaches the two-fluid system from a no-slip one-fluid system is used.
(c) The SIMPLE numerical procedure is extended to the two-phase flow in this paper. An iterative under-relaxation method of pressure correction for solving the system of difference equations of the two-fluid model of turbulent two-phase flow has been developed.
(d) The two-fluid model of turbulent two-phase flow has a great potential for further development. In particular, the establishment of a more rational mathematical model that can correctly relate the exchanges of mass, momentum and energy between two fluids is highly desirable.

6. Acknowledgments

This research is sponsored by the Hong Kong SAR Research Grants Council under Grant No. NSFC/HKU 8, the National Science Fund for Distinguished Young Scholars under Grant No. 50125924, the National Natural Science Foundation of China under Grant No. 49910161985, and the Research Fund for the Doctoral program of Higher Education under Grant No. 2000014112.

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